

New R package *arfurimaaparch* for Estimation of ARFURIMA-APARCH Model and Big Data Analytics

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Abstract

*This paper introduces the R package **arfurimaaparch** version 0.1.0 for time series computations, big data analytics and estimation of Autoregressive Fractional Unit Root Integral Moving Average-Asymmetric Power Autoregressive Conditional Heteroscedasticity (ARFURIMA-APARCH) model. The *fdr*, *arfurimaaparch*, *arfurimaaparchforecast*, *arfurimaaparchdiagnostic* and *arfurimaaparch.sim* are the main functions of the package. An improved version of the *arfurima* package version 1.1.0 of Jibrin and Rahman (2019) for implementing Monte Carlo simulation is also presented. Daily Nigeria all share index and West Texas Intermediate (WTI) crude oil prices for the period 26th January 2004 to 31st December 2018 were used to explained the usage of the packages. When the *arfurimaaparch* package is compared with other long memory packages, It would produce better stationary process after transformation, appropriate fractional differencing values in the interval of $1 < d < 2$, minimum Akaike Information Criteria values, larger log-likelihood values, minimum *p*-values of the ARFURIMA-APARCH parameters estimates and large *p*-values of the Ljung-Box, ARCH-LM and Jarque-Bera test. Findings show that both R packages and their functions are robust, simple and user-friendly. As conclusion, the R packages are suitable, good and reliable for time series analysis computations, statistical analysis and big data analytics.*

Keywords: Fractional unit root differencing, R, *arfurimaaparch* package, ARFURIMA-APARCH model and big data analytics.

1. Introduction

In the time series literatures, mean and hybrid models for modeling and forecasting time series were introduced (see Box and Jenkins (1976), Granger and Joyeux (1980), Hosking (1981), Porter-Hudak (1990), Beran (1999), Meerschaert *et al.* (2014), Boubaker *et al.* (2016), Pumi *et al.* (2019), Jibrin (2019) and Musa and Jibrin (2021)). Modifications of these models are important for handling current and future research challenges. One of the major issues with time series modeling is the availability of R packages for estimations and other analysis especially in the environment of R statistical software.

Development of new or modifying the existing algorithms and R packages would no doubt show the originality of work on these models. This could also indicate the clear contribution of a research work.

2. Related Works

Many R Packages developed in the literatures are synonyms to the targeted models and they could be classified as mini software (see R Package **arfima** in Veenstra (2012), **artfima** in McLeod *et al.*, (2016) and **arfurima** package of Jibrin and Rahman (2019)). There are other few packages that implement the mean, volatility and hybrid models. For example, the package **fracdiff** developed by Maechler (2020) for transforming time series that exhibits long memory, implementing fractional differencing and ARFIMA model estimation. The package **forecast** by Hyndman *et al.* (2020) also for ARFIMA modeling depends on **fracdiff** and thus carried over its deficiencies. The package **rugarch** by Ghalanos (2022) is for volatility and hybrid models.

Other packages developed for time series analysis are; **wmtsa** by William and Donald (2011) for wavelet methods and time series analysis; **LongMemoryTS** by Leschinski (2019) for long memory time series analysis; **nlstac** by Rodriguez-Arias *et al.* (2020) for fitting separable nonlinear models; **longmemo** by Beran *et al.*, (2020) for statistics and long memory processes; **mvtnorm** by Genz *et al.*, (2021) for multivariate time series modeling. In addition, R Core Team (2020), Borchers (2021) and Tsay *et al.*, (2022) are significant researches and contribution in the area of R package development.

In the literature, there are long memory packages developed for estimation of long memory mean models and their hybrid models. The packages regularly approximates the fractional difference value of a time series that supposed to be in the interval of $0 < d < 1$ to be $d = 0.5$. For example, the package **fracdiff** of Fraley *et al.* (2012) and the package **arfima** developed by Veenstra (2012). In addition, the **artfima** by McLeod *et al.*, (2016) exaggerates the fractional difference value of a times series greater than unity, $d > 2$. To the best of our knowledge and in the long memory time series literature, similar size of fractional differencing value were never discovered nor estimated using the available long memory estimation methods. The **rugarch** by Ghalanos (2022) for hybrid models estimation also restricted fractional difference value in the interval of $0 < d < 1$. In view of this, the current paper presents the R package **arfurimaaparch** for time series computations with fractional difference value assumed to be $1 < d < 2$, big data analytics and estimation of ARFURIMA-APARCH model. An improved version of the R package **arfurima** of Jibrin and Rahman (2019) that implements Monte Carlo simulation will be discussed.

3. Theoretical Review

The Fractional Unit Root Differencing (FURD) filter, ARFURIMA and ARFURIMA(p,d,q)-APARCH(s,v) will be discussed in this section.

3.1 The Fractional Unit Root Differencing Filter

All third level sectionheadings must be number as above, italicized, in Times New Roman and font size of 11. The content of the sections should be in the same font type and size.

Given $\{Y_t\}, t = 1, \dots, T$ that exhibit trend, interminable long memory and adjudged to be non-stationary, the fractional filter in ARFURIMA model for obtaining T_t is not evaluated using binomial expansion as used by Beran (1999), Meerschaert *et al.*, (2014) and Pumi *et al.*, (2019) but expanded by non-power operator given as:

$$T_t = \sum_{n=0}^N \{Y_{t-n} - Y_{t-n-1} - d^*(Y_{t-n} - Y_{t-n-2})Y_t\}. \quad (1)$$

where Y_t is the time series that is assumed to have fractional difference value $d \in (1,2)$ such that $d^* = d - 1, d^* \in (0,1)$ and both d and d^* are fractional differencing values.

3.2 ARFURIMA Model

The ARFURIMA model of Jibrin(2019) is defined as follows:

$$\phi(L)\{(1-L)(1-d^*(1+L))\}Y_t = \theta(L)\varepsilon_t, \tag{2}$$

where $\phi_i \geq 0$ for $i = 1, \dots, p$, $\theta_j \geq 0$ for $j = 1, \dots, q$ are the parameters of the ARFURIMA model. All the roots of the characteristics polynomials $\phi(L)$ and $\theta(L)$ lie outside the unit circle. $\{(1-L)(1-d^*(1+L))\}$ is the non-power fractional unit root differencing operator for the ARFURIMA mean model and the L is the backward shift operator.

3.3 ARFURIMA-APARCH Model

The ARFURIMA(p,d,q)-APARCH(s,v) of Jibrin *et al.*, (2022) given as:

$$X_t = (\phi(L)\{(1-L)(1-d^*(1+L))\})^{-1}\theta(L)[\omega + \alpha(L)(|\varepsilon_t| - \gamma\varepsilon_t)^\delta + \beta(L)\sigma_t^\delta]^{\frac{1}{\delta}}u_t. \tag{3}$$

In addition to all the parameter definitions of the ARFURIMA mean model, the $\alpha_r \geq 0$ for $r = 1, \dots, s$, $\beta_x \geq 0$ for $x = 1, \dots, v$, are parameters of the hybrid ARFURIMA-APARCH model. Similarly, all the roots of the characteristics polynomials $\beta(L)$ and $\alpha(L)$ lie outside the unit circle. γ is the leverage coefficient in the interval of $-1 < \gamma < 1$ and δ is the parameter for the power term that takes finite positive values. For detail derivation of the ARFURIMA and hybrid ARFURIMA-APARCH models see Jibrin (2019) and Jibrin *et al.*, (2022).

4. R Package Installation and Data

Before installing **arfurima** or **arfurimaaparch**, you must have installed R version 4.1.3 or higher, which is freely available at <http://www.r-project.org/>.

To install the R package **arfurima** or **arfurimaaparch** from a repository on your system named `source`, type the following in the R command prompt:

```
>install.packages("source/arfurima.tar.gz", repos = NULL, type = "source")
>install.packages("source/arfurimafiaparch.tar.gz", repos=NULL, type="source")
```

Then, install and activate the following R packages that are necessary for long memory mean and hybrid modeling .

```
>library(fracdiff);library(forecast); library(MASS);library(rugarch);
>library(archlmtest);library(tseries); library(fGarch)
```

Daily Nigeria all share index and West Texas Intermediate (WTI) crude oil prices for the period 26th January 2004 to 31st December 2018 were sourced from the Datastream database and used in this study.

The data is utilized in R as follows:

```
> nigindexwti<- read.csv(file.choose(), header=TRUE)
>names(nigindexwti)
[1] "date" "wti" "nigallsindex"
```

The data `nigindexwti` contain the date, wti and the `nigallsindex`.

5. The R Package **arfurima**

The improved version of the R package **arfurima** for time series computations, big data analytics and estimation of ARFURIMA model will be introduced and explain. It can be activated using the library function as:

```
>library(arfurima)
```

The R package **arfurima** comprised of `furd`, `arfurima`, `arfurimaforecast`, `arfurimadiagnostic` and `arfurima.sim` functions for the modeling of ARFURIMA model and simulation of FURI series. The improved version of the package is the implementation of the Monte Carlo simulation with replications.

It can be shown that the fractional difference parameter d for the data is greater than unity using the `fdGPH` function from package **fracdiff**:

```
> apply(nigindexwti[, -1], 2, fdGPH)

$wti$d
[1] 1.234138
$nigallsindex$d
[1] 1.210124
```

5.1 The *furd* function

To obtain the transformed fractional unit root differenced series T_t , use the filter in eq.(1) and applying the function `furd` found in package **arfurima**.

```
> head(apply(nigindexwti[, -1], 2, furd))

      wti nigallsindex
[1,] -0.4200000    -0.0800000
[2,] -0.1773721     1.296409
[3,] -0.5054236   -42.448673
[4,]  0.4100450    46.137929
[5,]  1.2393819   -9.293799
[6,] -0.9135039    26.350252
```

The both transformed series can be obtained as:

```
>furdiff_nigindexwti = apply(nigindexwti[, -1], 2, furd)
```

and graphical representation using:

```
>plot(ts(furdiff_nigindexwti), main=paste(" "))
```

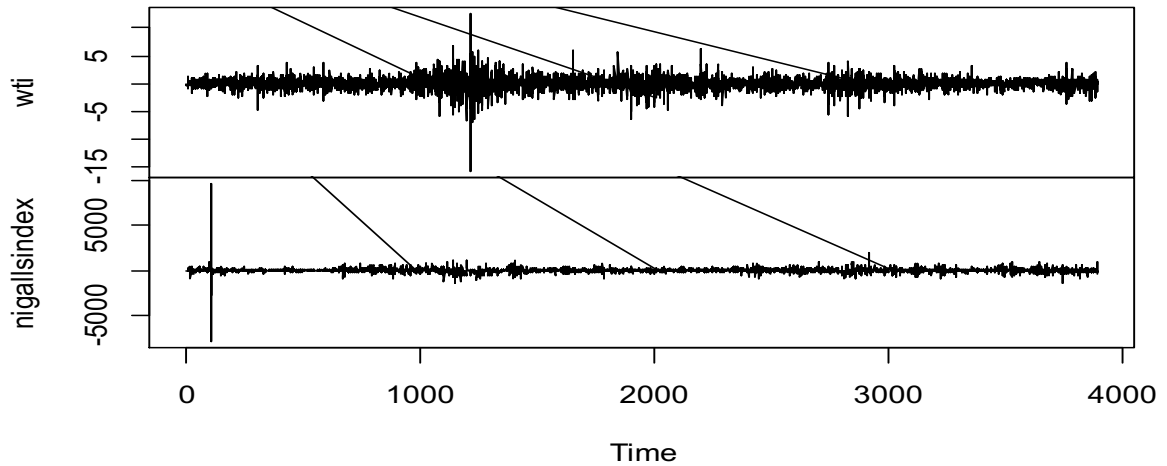


Figure 1. Returned filtered WTI and Nigeria All Share series using function furd

5.2 The arfurima function

The ARFURIMA model estimation for the series can be done by using the `arfurima` function in package `arfurima` as:

```
>d = fdGPH(nigindexwti$wti)$d
>arfurimawtill=arfurima(nigindexwti$wti,order=c(1,d,1))

$order
[1] 1.000000 1.234138 1.000000

$`Fractional difference`
      EST      SE  95%Lower  95%Upper
d.  1.0000000 0.0000000 1.0000000 1.0000000
d*  0.2341384 0.09095878 0.05586244 0.4124143
d   1.2341384 0.09095878 1.05586244 1.4124143

$Parameter
      EST      SE      z    Pr(>|z|)  95%Lower  95%Upper
ar 0.001299763 3.608525e-05 36.01924 0.0000 0.001229037 0.001370489
ma 0.426659527 6.501402e-04 656.25773 0.0000 0.425385275 0.427933778

$`Model adequacy`
      EST
Sigma 1.184333
logLik -6185.584
AIC    12375.17
```

```
attr("class")
[1] "arfurima"
```

The arfurima function returns the model order, the fractional difference estimate and its standard error, the parameter estimate and the model adequacy scores (AIC, sigma and log-likelihood).

5.3 The arfurimadiagnostic function

For the serial Ljung-Box, ARCH-LM and Jarque-Bera serial correlation analysis, we use the residuals from the fitted ARFURIMA model which can be obtained using the arfurimadiagnostic function as follows:

```
>arfurimawit11residuals= arfurimadiagnostic(arfurimawit11,plot=F)
>Box.test(arfurimawit11residuals,lag=5,type="Ljung-Box")
      Box-Ljung test
data:  arfurimawit11residuals
X-squared = 19.366, df = 5, p-value = 0.001642
>archlmtest(arfurimawit11residuals,lag=5)
      ARCH LM-test
Chi-squared = 733.92, df = 5, p-value < 2.2e-16
>jarque.bera.test(arfurimawit11residuals)
      Jarque Bera Test
data:  arfurimawit11residuals
X-squared = 18515, df = 2, p-value < 2.2e-16
```

The model diagnostic can be accessed using the function arfurimadiagnostic as:

```
> arfurimadiagnostic(arfurimawit11,plot=T)
```

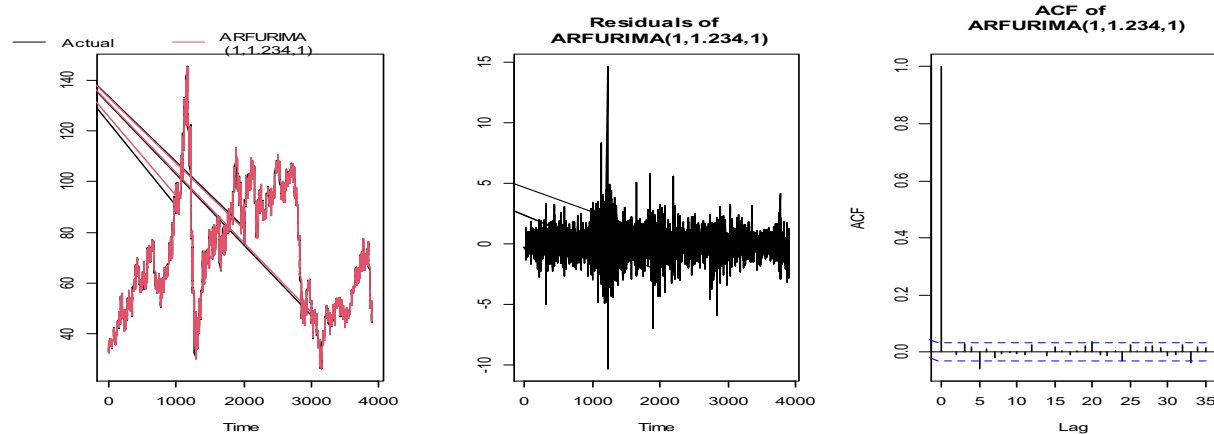


Figure 2: Diagnostic plots of ARFURIMA(1,1.234,1) Model Fitted to the WTI series.

5.4 The arfurimaforecast function

The model forecast can be obtained using the function arfurimaforecast as:

```
> arfurimaforecast(arfurimawtill, type="out.sample", h=10, plot=F)
```

| | Point Forecast | Lo 80 | Hi 80 | Lo 95 | Hi 95 |
|------|----------------|----------|----------|----------|----------|
| 3897 | 45.18305 | 43.20151 | 47.16458 | 42.15255 | 48.21355 |
| 3898 | 45.21945 | 42.50048 | 47.93842 | 41.06115 | 49.37775 |
| 3899 | 45.25797 | 41.95204 | 48.56391 | 40.20198 | 50.31396 |
| 3900 | 45.29830 | 41.48911 | 49.10748 | 39.47265 | 51.12394 |
| 3901 | 45.34016 | 41.08339 | 49.59692 | 38.83000 | 51.85032 |
| 3902 | 45.38335 | 40.71961 | 50.04709 | 38.25078 | 52.51592 |
| 3903 | 45.42770 | 40.38838 | 50.46701 | 37.72073 | 53.13466 |
| 3904 | 45.47306 | 40.08343 | 50.86269 | 37.23034 | 53.71579 |
| 3905 | 45.51933 | 39.80031 | 51.23835 | 36.77284 | 54.26582 |
| 3906 | 45.56640 | 39.53572 | 51.59709 | 36.34326 | 54.78954 |

The forecasts can also be plotted, for example the forecast of next 180 days can be obtained as:

```
> arfurimaforecast(arfurimawtill, type="out.sample", h=180, plot=T)
```

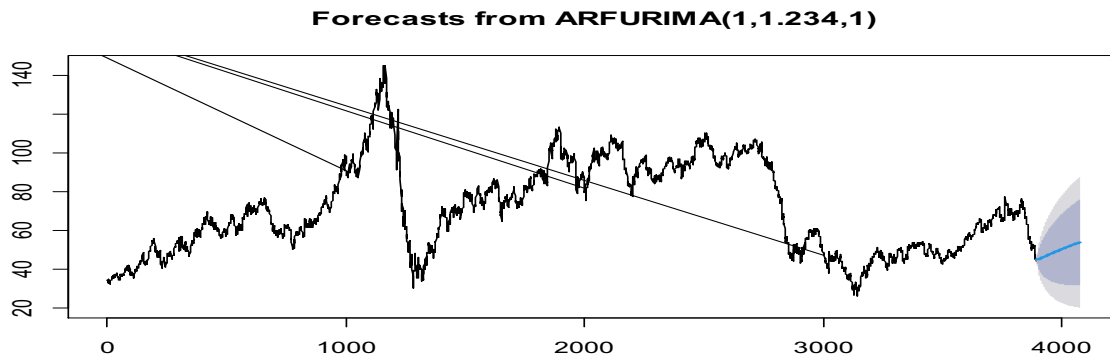


Figure 3. Forecast of next 180 days for WTI indices.

5.5 The arfurima.sim function

The Monte Carlo simulation is the improved version of the R package **arfurima** of the ARFURIMA model with replications and can be done using the function `arfurima.sim(n, phi, d, theta, replication)` as:

```
> simFURI=arfurima.sim(1000, 0.4, 1.6, 0.5, 500)
> d=fdGPH(simFURI) $d
> d
[1] 1.58611
```

The simulated 1000 FURI series with fractional differencing values in the intervals of $1 < d < 2$ can also be plotted as:

```
> ts.plot(simFURI)
```

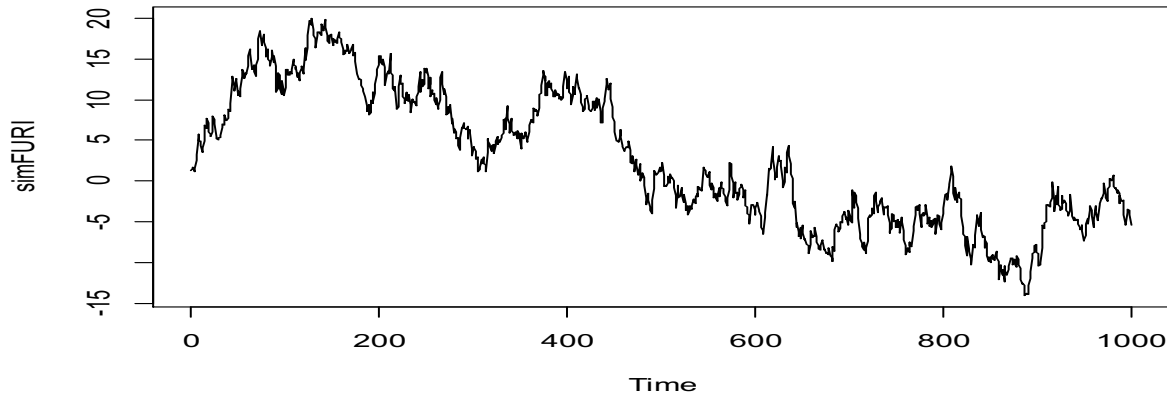


Figure 4: Generated 1000 FURI Series Based on 500 Replications Using the ARFURIMA Model.

6. The R Package **arfarfurimaaparch**

The `fdr`, `arfarfurimaaparch`, `arfurimaaparchforecast`, `arfurimaaparchdiagnostic` and `arfurimaaparch.sim` are the major functions of **arfurimafiaparch** package for time series computations, big data analytics and implementation of fractional unit root differencing filter in (1), estimation of the hybrid ARFURIMA-APARCH model in eq. (3), forecasts using hybrid ARFURIMA-APARCH model, producing diagnostic plots of hybrid ARFURIMA-APARCH model and Monte Carlo simulation using hybrid ARFURIMA-APARCH model respectively.

```
>library(arfurimaaparch)
```

6.1 The `fdr` function

The above is the first six observations from the transformation by the proposed `furd` non-power fractional operator function. In the same vein, we can obtain the fractional difference returns by using the `fdr` function as:

```
> head(apply(nigindexwti[, -1], 2, fdr))
```

```
      wti  nigallsindex
[1,] -1.2280856 -0.0003526245
[2,] -0.5279351  0.0057141268
[3,] -1.5246136 -0.1873109518
[4,]  1.2383527  0.2036230588
[5,]  3.6381560 -0.0410188290
[6,] -2.6631717  0.1160998625
```

6.2 The `arfurimaaparch` function

The distribution of error for the hybrid ARFURIMA-APARCH model within the subroutine of **arfurimaaparch** can also be specified. The distributions the distributions in the R syntax are indicated as `error.dist="norm"`, `"std"`, `"ged"` and `"sstd"` for Gaussian, Student-t, Generalized Error Distributions, and Skew Student-t respectively. The ARFURIMA-APARCH model can be obtained as:


```
>hybridsstd11=arfurimaaparch(nigindexwti$nigallsindex,mean.order =
  c(1,1),var.order=c(1,1),var.model="APARCH",error.dist="sstd")

>hybridsstd11

$`Variance Model`
[1] "APARCH"
$`Error Distribution`
[1] "sstd"
$mean.order
[1] 1 1
$var.order
[1] 1 1
$`Time Series Long Memory Value`
[1] 1.210124

$Parameter
      Par. Est.   Std. Errors   t-stat. Pr(>|t|)   95%Lower   95%Upper
mu      6.42035405 0.0025304110 2537.2772 0.0000   6.41539454 6.4253136
ar1     0.96951699 0.0009812258  988.0672 0.0000   0.96759382 0.9714402
ma1    -0.94963344 0.0009742731 -974.7097 0.0000  -0.95154298 -0.9477239
omega  31.38895656 0.0055981395 5607.0336 0.0021  31.37798441 31.3999287
alpha1  0.30309292 0.0005503658  550.7117 0.0000   0.30201423 0.3041716
beta1   0.73767985 0.0008612074  856.5647 0.0000   0.73599192 0.7393678
gamma1 -0.03726068 0.0001928039 -193.2569 0.0000  -0.03763857 -0.0368828
delta   1.12911588 0.0010608043 1064.3960 0.0002   1.12703674 1.1311950
skew    1.06428736 0.0010277756 1035.5250 0.0001   1.06227296 1.0663018
shape   3.47098584 0.0018660573 1860.0640 0.0001   3.46732843 3.4746432

$`Model adequacy`
      Estimates
logLik -26060.59133
AIC      13.38326

attr(,"class")
[1] "arfurimaaparch"
```

6.3 The arfurimaaparchdiagnostic function

The serial correlation analysis of the residuals of the fitted ARFURIMA(1,1)-APARCH(1,1) model is obtained using the arfurimaaparchdiagnostic function as follows:

```
>residualsnigallsindex2msstd11=arfurimaaparchdiagnostic(nigallsindex2msstd11
,plot=F)

>Box.test(residualsnigallsindex2msstd11,lag=5,type="Ljung-Box")

Box-Ljung test
```

```
data: residuals nigallsindex2msstd11
X-squared = 9.3359, df = 5, p-value = 0.09639

> archlmtest(residuals nigallsindex2msstd11, lag=5)

ARCH LM-test

Chi-squared = 7.7954, df = 5, p-value = 0.1679

The model diagnostic can be accessed using the function arfurimaaparchdiagnostic as:

> arfurimaaparchdiagnostic(hybridsstd11, plot=T)
```

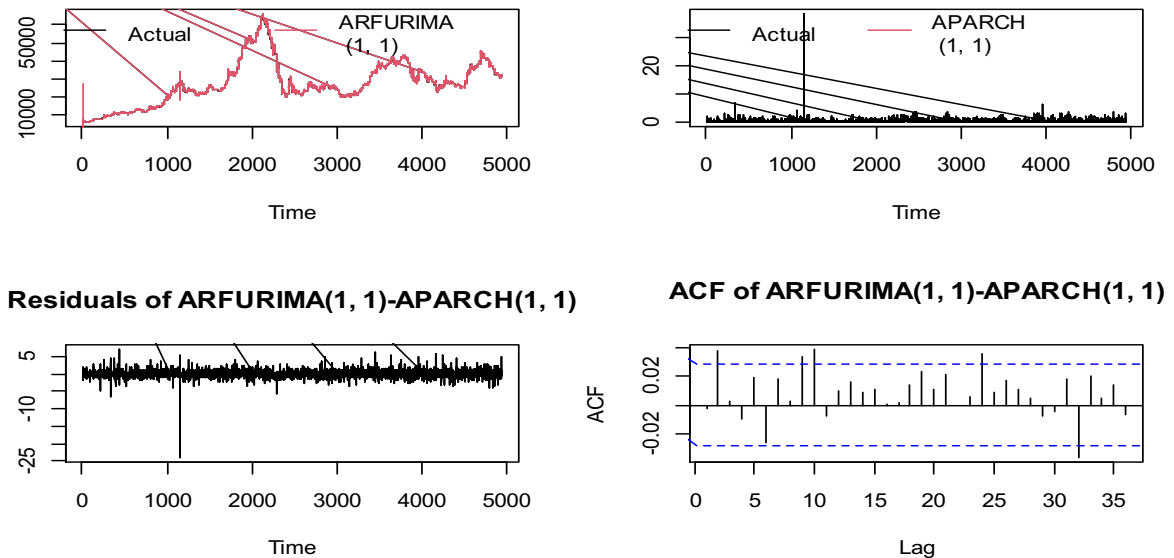


Figure 5: Diagnostic plot for the ARFURIMA-APARCH Fitted to Nigeria All Share Index.

6.4 The arfurimaaparchforecast function

The hybrid ARFURIMA(1,1.2,1)-APARCH(1,1) model forecasts can be obtained using the function arfurimaaparchforecast as:

```
> arfurimaaparchforecast(hybridsstd11, type="out.sample", h=10, plot=F)

      yt      sigma
T+1 31586.32 481.0106
T+2 31586.25 487.1157
T+3 31586.18 493.1083
T+4 31586.10 498.9933
T+5 31586.03 504.7758
T+6 31585.96 510.4601
```

```
T+7 31585.89 516.0505
T+8 31585.81 521.5507
T+9 31585.74 526.9645
T+10 31585.67 532.2951
```

```
> arfurimaaparchforecast(hybridsstd11, type="out.sample", h=10, plot=T)
```

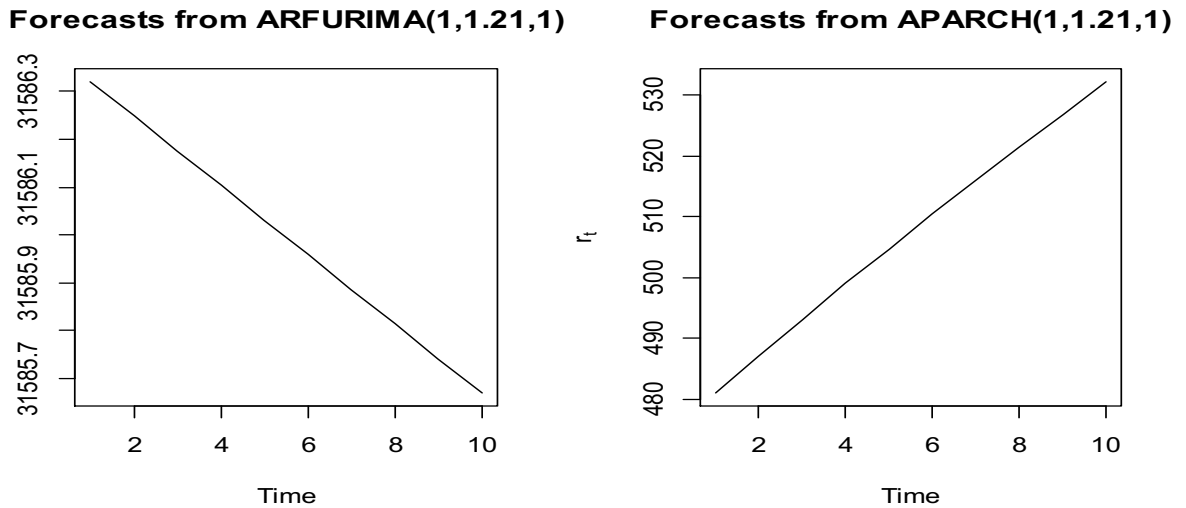


Figure 6: Forecasts of Mean and Volatility Components for ARFURIMA(1,1)-APARCH(1,1) Using Nigeria All Share Index.

6.5 The *arfurimaaparch.sim* function

The Monte Carlo simulation using the hybrid ARFURIMA-APARCH model with replications can be done using the function `arfurimaaparch.sim(n, phi, d, theta, alpha, beta, gamma, delta, replication)` as:

```
> hybridSIM=arfurimaaparch.sim(n=1000, phi=0.5, d=1.2, theta=0.3, alpha=0.4, beta=
= 0.2, gamma=0.8, delta=1.2, replication=100)
> plot(ts(hybridSIM), ylab = "Simulated ARFURIMA-APARCH Series")
```

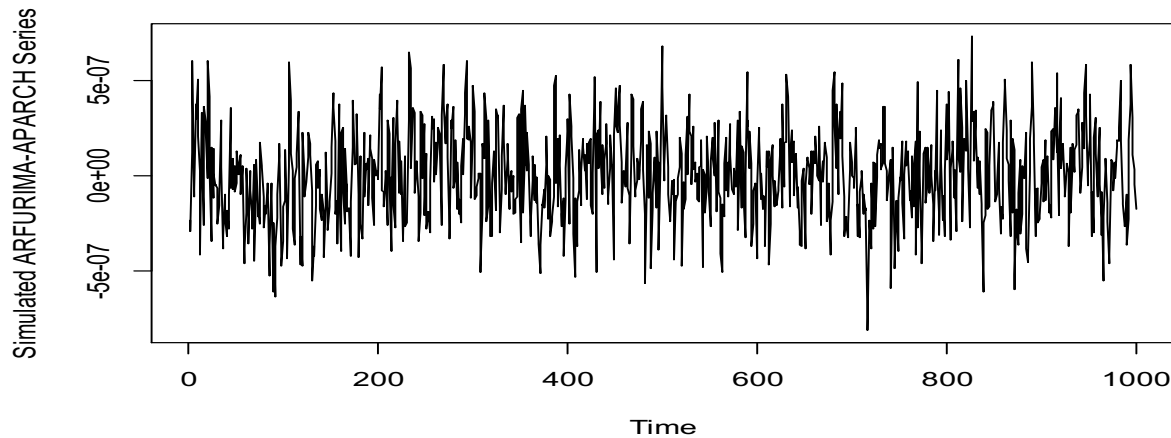


Figure 6: Generated 1000 Series Based on 100 Replications Using the ARFURIMA-APARCH Model.

7. Conclusion

In this study, the R package **arfurimaaparch** version 0.1.0 was developed for time series computations, big data analytics and estimation of hybrid ARFURIMA-APARCH model. Also, an improved R package **arfurima** for modeling of ARFURIMA model was introduced. The `fdr`, `arfurimaaparch`, `arfurimaaparchdiagnostic`, `arfurimaaparchforecast` and `arfurimaaparch.sim` are the major functions of the R package **arfurimaaparch** version 0.1.0. The Monte Carlo simulation of FURI series with replications is the improved version of the R package **arfurima** version 1.1.0. The WTI and Nigeria all share indexes used in the study were transformed using both `furd` and `fdr` functions and both produces a stationary process. In addition, ARFURIMA-APARCH models were estimated using `arfurimaaparch` function. After hybrid ARFURIMA-APARCH model estimation, the time series computations and big data analytic which formed part of the serial correlation analysis and forecasts were carried out by using the `arfurimaaparchdiagnostic` and `arfurimaaparchforecast`. Also, the diagnostic and forecast plots were obtained using the `arfurimaaparchdiagnostic` and `arfurimaaparchforecast` functions respectively. Finally, the `arfurimaaparch.sim` function based on 100 replications generates 1000 volatile series. Overall, the results show that the R package **arfurimaaparch** and its associated functions are robust, user-friendly, simple and easy to apply. This is because the package possessed the qualities of producing good diagnostic test graphs as shown in 3 in 1 and 4 in 1 in Figure 2 and Figure 5 respectively. Also, its robustness can be seen since the residuals of the estimated ARFURIMA mean models can be assumed to be normal, student-t, skew-student and generalized error distributions as shown in sub-section 6.2. Again, large time series can be simulated with replications using the package. In addition, its user-friendly qualities and simplicity can be traced to the short line of codes used in all sections of the manuscript. When the **arfurimaaparch** package is compared with other long memory packages for example **rugarch** package, It would produce better stationary process after transformation, appropriate fractional differencing values in the interval of

$1 < d < 2$, minimum Akaike Information Criteria values, larger log-likelihood values, minimum p-values of the ARFURIMA-APARCH parameters estimates and large p-values of the Ljung-Box, ARCH-LM and Jarque-Bera test. This indicates that the R package **arfurimaaparch** is sophisticated, suitable and reliable for time series computations, big data analytics and estimation of the hybrid ARFURIMA-APARCH models. Future works should focus on developing R packages for time series computations, big data analytics and estimation of existing mean, volatility and hybrid models.

8. References

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