

# Analysis of Multi-Robot Cooperative Hunting Using Differential Game Technique

Bashir Abdullahi Baba<sup>1</sup>, Salisu Garba<sup>2</sup>, and Nasir Faruk<sup>3</sup>

<sup>1</sup>Department of Cyber Security, Sule Lamido University, Kafin Hausa

<sup>2</sup>Department of Software Engineering, Sule Lamido University, Kafin Hausa

<sup>3</sup>Department of Information Technology, Sule Lamido University, Kafin Hausa  
abashir.abdullahibaba@slu.edu.ng<sup>1</sup>, bsalisu.garba@slu.edu.ng<sup>2</sup>, faruk.n@slu.edu.ng<sup>3</sup>

## Abstract

*This research presents two robots cooperative hunting behavior by the use of a differential game method. There are two pursuer robots that are trying to find and then surround another robot prey evader. The purpose of the game is for the two pursuer robots to find the evader simultaneously or when one of the robots finds the evader first it will wait for the other robot to cooperate with it within a specified period of time. We use differential game theory to formulate the problem with a system of an ordinary differential equation. The conditions for the termination of the game were given to be when one of the pursuers catch the evader and the other cooperate with it or when the two pursuers catch it simultaneously. We carefully analyzed all the mathematical equations developed using the ordinary differential equations for the game and present the sufficient conditions that can warrant the two pursuers to catch the evader either simultaneously or one first and then the other cooperates with the first one which will result in termination of the game. We also show mathematically that in the course of the game the robot prey evader tries to prolong the capture time, while the pursuer robots try to shorten the capture time.*

**Keywords:** *Differential Game, Robot, Hunting Game, Pursuer, Evader*

## 1. Introduction

Multi-robot system is a system comprises of two or more robots acting together towards a desired goal. These kinds of systems are designed to act in such a way that the group's common objective is achieved and they are controlled by using cooperative control laws. In multi-agent system analysis modeling the cooperative behavior of the multi-robot is very critical (Schaal & Atkeson, 2010; Stone & Veloso, 2000). Two models are very important, the first one is a mathematical model that gives the computer-based simulations of all the necessary properties of the robot, and the other model which will give all the necessary tools used in the analysis of multi-robot system and also used to maintain the most important aspects of the system in terms of operation. Up till now, most of the researchers are devoted to the area of implementing the real systems. Experiments and simulations are continuously used in a defined context to improve the control strategies until a defined task is accomplished successfully by the robot systems (Nehmzow, 2006). Generally speaking, the above methods can only be used to show that the robot systems can meet a given task. But the control laws may not be usually implemented successfully on platform of another robot or when the environmental conditions are changed. Considering the real systems, the manner of interactions between robots and its environment is still very difficult to understand. Similarly, mathematics methods cannot be used to model the behavior of robots' systems, and analyzing the behavior quantitatively is difficult (Varghese & McKee, 2009).

In real world, the abstract of the multi-robot systems can be found from the mathematical models of the cooperative behaviors of robots (Nighot et al., 2012). The research of the mathematical models gives better understanding of the robots' interaction with the environment (Helbing, 2010). There is many research work that try to model the cooperative behaviors of robot (Davoodi et al., 2021; Gualtieri et al., 2021; Parsa & Saadat, 2021;

Teslya et al., 2021). These models help in comprehending what is the main components in the behaviors of the system, and at the same time it can help us to improve our understanding in the properties which can emerge from the collective behaviors of the robot. A method that deals with control and perception in amidst of uncertainty is known as probabilistic robotics (Moulin-Frier & Oudeyer, 2013). Thrun, (2000) presented the use of probabilistic robotics method for programming robots, the method generally performs better in complex real-world applications than all the other approaches that neglect the uncertainty of the robot. Burgard et al., (2000) presented the use of the probabilistic method in case of coordinating multiple robots, in this method all the utilities for the target points as well as all the costs for reaching the target point were all put in to consideration. Fox et al. in Fox et al., (2000) presented an algorithm for the localization of mobile robots' collaboration. In their method they were able to use the Markov localization type-based sample in localizing the mobile robots at any given time.

Differential game is one of the most popular areas in applied mathematics and is still having more perspectives for researchers due to its wide usage in finding the solution of most of the real-life problems encountered in many areas such as engineering, economics, behavioral biology, missile guidance, etc. The study of differential game was founded by Rufus Isaacs (Vajda, 1967), in which "the homicidal chauffeur game" is among what were analyzed then. (L. Berkovitz, 1974; L. D. Berkovitz, 1967; Pontryagin, 1988; Rikhsiev, 1989; Subbotin & Chentsov, 1981; Vajda, 1967) give the basic principles of differential games.

There are many classifications of differential game problems in literature, among which is what is known as the pursuit-evasion differential game. This type of the differential game consists of two players which have a contradictory aim, the players are called evader and pursuer. The pursuer's aim is to finish the game in a finite time, while the evader's aim is to prolong the time of the game. These strategies are useful in many areas, such as coastguard saving shipwrecked sailors, missile launched at enemy aircraft, etc. Differential games that consist of more than one pursuer with geometrical and integral constrained were studied in (Azamov & Samatov, 2000; Garcia et al., 2021; Grinikh & Petrosyan, 2021; G. Ibragimov et al., 2018; G. I. Ibragimov, 2003, 2005; G. I. Ibragimov et al., 2011, 2012; Leong & Ibragimov, 2008; Subbotin & Chentsov, 1981).

In this paper, we study the use of differential game method in a multi-robots cooperative hunting game, in which two of the robots ( $R_1$  and  $R_2$ ) were the pursuers and the other robot ( $E$ ) prey was an evader. The main objective of this game is to have the condition in which the two of the pursuers can catch the evader at the same time or if one of the pursuers catch it first, then it will wait until the other pursuer cooperate within a specified period of time otherwise if the time limit elapses a fresh searching will start. We were able to mathematically represent the problem by forming an ordinary differential equation and also, we analyzed the equations by showing and proving the all the condition needed to stop/end the game. All the necessary conditions to be satisfied in the game was given and proved.

## 2. Formulation of the problem

For this study, the multi-robot system comprises of similar robots interrelated together and working with each other towards the same objective. These robots are designated as  $P_1$  and  $P_2$  that is the pursuers, which are pursuing a single robot prey (the evader)  $E$ . In multi robot system the most challenging task is the cooperative pursuit this is because the accomplishment of this task has implications in the behavior of multi-robot collaborations.

### 2.1 Hunting method

The method of hunting consists of the following behaviors namely: searching, following, and surrounding. The pursuer team consists of 1-robot team and 2-robots team. At first, the pursuers ( $R_1$  and  $R_2$ ) will be searching the evader ( $E$ ) by the use of searching devices or sensors, this stage is called searching. Secondly if the evader ( $E$ ) come into region of detecting, then the pursuer robots ( $R_1$  and  $R_2$ ) will use their tracking algorithms to follow it, this stage is called following. The pursuers ( $R_1$  and  $R_2$ ) will surround the evader ( $E$ ) whenever it is possible by moving to the immediate surroundings of the evader, this stage is called surrounding. This will continuously be repeated until the evader is captured.

Once a single pursuer sees the evader, it will follow it, making 1-robot team and it will then wait for the other robot pursuer to cooperate. If the other pursuer is unable to detect the 1-robot team within a specified period of time, then the robot pursuer that made that 1-robot team will give up the target and start searching for another evader randomly. While if the other pursuer robot is able to detect that same evader with 1-robot team, then the team will change to 2-robots team.

### 2.2 Mathematical formulation of the problem

Let assume that the simple motion dynamics and constant speed of the robots  $E, R_1$  and  $R_2$  were given as  $S_E, S_{R_1}$  and  $S_{R_2}$  respectively. And also, the controls of  $E, R_1$  and  $R_2$  were given as their respective instantaneous headings  $\phi, \Psi_1$ , and  $\Psi_2$ . Then, we can defined the states of  $E, R_1$  and  $R_2$  using their respective Cartesian coordinates for realistic plane as:  $X_E = (x_E, y_E)$ ,  $X_{R_1} = (x_{R_1}, y_{R_1})$  and  $X_{R_2} = (x_{R_2}, y_{R_2})$  respectively. Therefore, we can say that the complete hunting state can be given as:  $X = (x_E, y_E, x_{R_1}, y_{R_1}, x_{R_2}, y_{R_2}) \in \mathbb{R}^6$ .

Since  $R_1$  and  $R_2$  can affect the state of the hunting by simply selecting states from their instantaneous respective headings  $\Psi_1$ , and  $\Psi_2$ , then our control variables of the pursuers and evader are their instantaneous heading angles, given as:  $U_R = \{\Psi_1, \Psi_2\}$  and  $U_E = \{\phi\}$  respectively. In the realistic plane  $\dot{X} = f(x, U_R, U_E)$  the following ordinary differential equations system gives the normalized dynamics:

$$\begin{aligned} \dot{x}_E &= \cos \phi, & \dot{y}_E &= \sin \phi \\ \dot{x}_{R_1} &= \beta_1 \cos \Psi_1, & \dot{y}_{R_1} &= \beta_1 \sin \Psi_1 \\ \dot{x}_{R_2} &= \beta_2 \cos \Psi_2, & \dot{y}_{R_2} &= \beta_2 \sin \Psi_2 \end{aligned} \tag{1}$$

Where,  $\beta_1 = \frac{S_{R_1}}{S_E} > 1$  and  $\beta_2 = \frac{S_{R_2}}{S_E} > 1$  are the normalized speeds for the two pursuers (robots) (note: their values may differ)

If the initial state can be given as:

$$X_0 = (x_{E0}, y_{E0}, x_{R_{10}}, y_{R_{10}}, x_{R_{20}}, y_{R_{20}}) = X(t_0)$$

Then, the hunting (game) ends at  $t = t_f$  which corresponds to the state of the system that satisfies one of the following:

$$\begin{aligned} \text{i) } &x_{R_1} = x_E, \quad y_{R_1} = y_E, \text{ or} \\ \text{ii) } &x_{R_2} = x_E, \quad y_{R_2} = y_E \text{ or} \\ \text{iii) } &\text{Both i) \& ii)} \end{aligned} \tag{2}$$

Hence, the state for the termination is given as:

$$X_f = (x_{Ef}, y_{Ef}, x_{R_{1f}}, y_{R_{1f}}, x_{R_{2f}}, y_{R_{2f}}) = X(t_f)$$

Since the pursuers are trying to shorten the capture time, and at the same, the evader is trying to prolong time the capture time, hence the performance function will be:

$$\min_{\Psi_1, \Psi_2} \max_{\phi} \int_0^{t_f} dt \tag{3}$$

### 3. Analysis of the hunting (game)

Here, we give our co-states as follows:

$$\lambda = (\lambda_{x_E}, \lambda_{y_E}, \lambda_{x_{R_1}}, \lambda_{y_{R_1}}, \lambda_{x_{R_2}}, \lambda_{y_{R_2}}) \in \mathbb{R}^6$$

Hence, the Hamiltonian ( $H$ ) for our differential game can be given as:

$$H = 1 + \lambda_{x_E} \cos \phi + \lambda_{y_E} \sin \phi + \beta_1 \lambda_{x_{R_1}} \cos \Psi_1 + \beta_1 \lambda_{y_{R_1}} \sin \Psi_1 + \beta_2 \lambda_{x_{R_2}} \cos \Psi_2 + \beta_2 \lambda_{y_{R_2}} \sin \Psi_2 \quad (4)$$

Therefore, we can obtain the optimal control inputs from the following:

$$\min_{\Psi_1, \Psi_2} \max_{\phi} H \equiv 0 \quad (5)$$

The optimal controls headings  $E, R_1$  and  $R_2$  can be characterized by the following:

$$\begin{aligned} \cos \phi^* &= \frac{\lambda_{x_E}}{\sqrt{\lambda_{x_E}^2 + \lambda_{y_E}^2}}, & \sin \phi^* &= \frac{\lambda_{y_E}}{\sqrt{\lambda_{x_E}^2 + \lambda_{y_E}^2}} \\ \cos \Psi_1^* &= -\frac{\lambda_{x_{R_1}}}{\sqrt{\lambda_{x_{R_1}}^2 + \lambda_{y_{R_1}}^2}}, & \sin \Psi_1^* &= -\frac{\lambda_{y_{R_1}}}{\sqrt{\lambda_{x_{R_1}}^2 + \lambda_{y_{R_1}}^2}} \\ \cos \Psi_2^* &= -\frac{\lambda_{x_{R_2}}}{\sqrt{\lambda_{x_{R_2}}^2 + \lambda_{y_{R_2}}^2}}, & \sin \Psi_2^* &= -\frac{\lambda_{y_{R_2}}}{\sqrt{\lambda_{x_{R_2}}^2 + \lambda_{y_{R_2}}^2}} \end{aligned} \quad (6)$$

And also, we can get the co-state dynamic by using:

$$\dot{\lambda} = -\frac{\partial H}{\partial x} \quad (7)$$

$$\Rightarrow \dot{\lambda}_{x_E} = \dot{\lambda}_{y_E} = \dot{\lambda}_{x_{R_1}} = \dot{\lambda}_{y_{R_1}} = \dot{\lambda}_{x_{R_2}} = \dot{\lambda}_{y_{R_2}} = 0 \quad (8)$$

This implies that, the robots' optimal trajectories are straight lines, because all the co-states and the optimal control inputs are constants.

Since our concern in this research is the termination condition in 2 (iii) above, that is a case where both robot 1 ( $R_1$ ) and robot 2 ( $R_2$ ) capture the evader ( $E$ ), then we have the following conditions to be satisfied simultaneously, i.e.

$$x_{R_{10}} + \beta_1 t_{f1} \cos \Psi_1^* = x_{E0} + t_{f1} \cos \phi^* \quad (9)$$

$$y_{R_{10}} + \beta_1 t_{f1} \sin \Psi_1^* = y_{E0} + t_{f1} \sin \phi^*$$

where:

$$t_{f1} = c_1 \cos(\phi^* - \lambda_1) + \sqrt{c_1^2 \cos^2(\phi^* - \lambda_1) + c_1 r_{10}}$$

$$\lambda_1 = \tan^{-1} \left( \frac{y_{E0} - y_{R_{10}}}{x_{E0} - x_{R_{10}}} \right)$$

$$r_{10} = \sqrt{(x_{E0} - x_{R_{10}})^2 + (y_{E0} - y_{R_{10}})^2}$$

$$c_1 = \frac{1}{\beta_1^2 - 1} r_{10}$$

and also,

$$x_{R_{20}} + \beta_2 t_{f2} \cos \Psi_1^* = x_{E0} + t_{f2} \cos \phi^* \quad (10)$$

$$y_{R_{20}} + \beta_2 t_{f2} \sin \Psi_2^* = y_{E0} + t_{f2} \sin \phi^*$$

where:

$$t_{f2} = c_2 \cos(\phi^* - \lambda_2) + \sqrt{c_2^2 \cos^2(\phi^* - \lambda_2) + c_2 r_{20}}$$

$$\lambda_2 = \tan^{-1}\left(\frac{y_{E0} - y_{R20}}{x_{E0} - x_{R20}}\right)$$

$$r_{20} = \sqrt{(x_{E0} - x_{R20})^2 + (y_{E0} - y_{R20})^2}$$

$$c_2 = \frac{1}{\beta_2^2 - 1} r_{20}$$

Since  $\beta_1, \beta_2 > 1$  there exist unique solutions  $\Psi_1$  &  $\Psi_2$  for  $\phi \in [0, 2\pi]$ . In the case when  $t_{f1} = t_{f2} = 0$ , that is  $r_1 = r_2 = 0$ , the evader has been captured by both robot 1 ( $R_1$ ) and robot 2 ( $R_2$ ) and the game (hunting) is over.

In this situation the triple  $(\phi, \Psi_1, \Psi_2)$  which satisfies (9) & (10) simultaneously with  $t_{f1} = t_{f2}$  will be gotten by using the point where the two Apollonius circles meet. The circles are based on  $r_{10}$  with speed ratio  $\beta_1$  and  $r_{20}$  with speed ratio  $\beta_2$  respectively. The center of the circles between  $E$  &  $R_1$  and  $E$  &  $R_2$  is at distance

$$c_1 = \frac{1}{\beta_1^2 - 1} r_{10} \text{ \& } c_2 = \frac{1}{\beta_2^2 - 1} r_{20} \text{ from } E$$

and their radii are

$$\rho_1 = \frac{\beta_1}{\beta_1^2 - 1} r_{10} \text{ and } \rho_2 = \frac{\beta_2}{\beta_2^2 - 1} r_{20} \text{ respectively.}$$

The coordinates of the circles are:

$$\begin{aligned} a_1 &= x_E + \rho_1 \cos \lambda_1, & b_1 &= y_E + \rho_1 \sin \lambda_1 \\ a_2 &= x_E + \rho_2 \cos \lambda_2, & b_2 &= y_E + \rho_2 \sin \lambda_2 \end{aligned}$$

Hence, the equation of the circles is:

$$(x - a_1)^2 + (y - b_1)^2 = \rho_1^2 \tag{11}$$

$$\text{and } (x - a_2)^2 + (y - b_2)^2 = \rho_2^2 \text{ respectively.}$$

The point of intersection of the two circles guarantees simultaneous interception by both robots ( $R_1$  &  $R_2$ ). Hence for this capture, we have:

$$\lambda_m \leq \phi_s^* \leq \lambda_M$$

where:

$$\lambda_m = \min\{\lambda_1, \lambda_2\} \text{ and } \lambda_M = \max\{\lambda_1, \lambda_2\}.$$

Consequently, we have the following proposition.

**Proposition:**

Considering the 2-pursuer 1-evader Robot cooperative hunting game. The solution when both Robots capture the evader can be given as:

$$\phi^* = \phi_s^*, \Psi_1^* = \Psi_{1s}^*, \Psi_2^* = \Psi_{2s}^*$$

when:

$$t_{f1}(\lambda_1) = t_{f2}(\lambda_1)$$

and

$$t_{f1}(\lambda_2) = t_{f2}(\lambda_2)$$

#### 4. Conclusion

Two robots hunting game was discussed and analysed by the use of differential game method. The robot collective behaviours' evolutions represent the transition between distinct states of robots. The method partitioned the state space in to three distinct regions. The two robots hunting game consists of two pursuer robots and a single evader robot. The game ended when the two pursuers catch the evader simultaneously, ordinary differential equation was used to represent and analyzed the system. All the necessary and required condition was mathematically shown and proved and also the solution when the two robots (pursuers) capture the evader robot was provided.

#### References

- Azamov, A. A., & Samatov, B. (2000).  $\pi$ -strategy. An elementary introduction to the theory of differential games. *Taskent: National Univ. of Uzb.*
- Berkovitz, L. (1974). Differential games. *IEEE Transactions on Automatic Control*, 19(5), 630–631.  
<https://doi.org/10.1109/TAC.1974.1100627>
- Berkovitz, L. D. (1967). *A survey of differential games, mathematical theory of control*, eds. AV Balakrishnan and LW Neustadt. Academic Press, New York.
- Burgard, W., Moors, M., Fox, D., Simmons, R., & Thrun, S. (2000). Collaborative multi-robot exploration. In *Proceedings - IEEE International Conference on Robotics and Automation* (Vol. 1, pp. 476–481). IEEE.  
<https://doi.org/10.1109/robot.2000.844100>
- Davoodi, M., Faryadi, S., & Velni, J. M. (2021). A Graph Theoretic-Based Approach for Deploying Heterogeneous Multi-agent Systems with Application in Precision Agriculture. *Journal of Intelligent and Robotic Systems: Theory and Applications*, 101(1). <https://doi.org/10.1007/s10846-020-01263-4>
- Fox, D., Burgard, W., Kruppa, H., & Thrun, S. (2000). Probabilistic approach to collaborative multi-robot localization. *Autonomous Robots*, 8(3), 325–344. <https://doi.org/10.1023/A:1008937911390>
- Garcia, E., Casbeer, D. W., Von Moll, A., & Pachter, M. (2021). Multiple Pursuer Multiple Evader Differential Games. *IEEE Transactions on Automatic Control*, 66(5), 2345–2350.  
<https://doi.org/10.1109/TAC.2020.3003840>
- Grinikh, A. L., & Petrosyan, L. A. (2021). AN EFFECTIVE PUNISHMENT for AN n-PERSON PRISONER'S DILEMMA on A NETWORK. *Trudy Instituta Matematiki i Mekhaniki UrO RAN*, 27(3), 256–262.  
<https://doi.org/10.21538/0134-4889-2021-27-3-256-262>
- Gualtieri, L., Rauch, E., & Vidoni, R. (2021). Emerging research fields in safety and ergonomics in industrial collaborative robotics: A systematic literature review. *Robotics and Computer-Integrated Manufacturing*, 67, 101998. <https://doi.org/10.1016/j.rcim.2020.101998>
- Helbing, D. (2010). Quantitative sociodynamics: Stochastic methods and models of social interaction processes. In *Quantitative Sociodynamics: Stochastic Methods and Models of Social Interaction Processes* (pp. 1–333). Springer Berlin Heidelberg. <https://doi.org/10.1007/978-3-642-11546-2>
- Ibragimov, G., Ferrara, M., Kuchkarov, A., & Pansera, B. A. (2018). Simple Motion Evasion Differential Game of Many Pursuers and Evaders with Integral Constraints. *Dynamic Games and Applications*, 8(2), 352–378.  
<https://doi.org/10.1007/s13235-017-0226-6>
- Ibragimov, G. I. (2003). Collective pursuit with integral constraints on the controls of players. *Matematicheskie Trudy*, 6(2), 66–79.
- Ibragimov, G. I. (2005). Optimal pursuit with countably many pursuers and one evader. *Differential Equations*, 41(5), 627–635. <https://doi.org/10.1007/s10625-005-0198-y>
- Ibragimov, G. I., Azamov, A. A., & Khakestari, M. (2011). Solution of a linear pursuit-evasion game with integral constraints. *ANZIAM Journal*, 51, 59. <https://doi.org/10.21914/anziamj.v52i0.3605>
- Ibragimov, G. I., Salimi, M., & Amini, M. (2012). Evasion from many pursuers in simple motion differential game with integral constraints. *European Journal of Operational Research*, 218(2), 505–511.  
<https://doi.org/10.1016/j.ejor.2011.11.026>
- Leong, W. J., & Ibragimov, G. I. (2008). A multiperson pursuit problem on a closed convex set in Hilbert space. *Far East Journal of Applied Mathematics*, 33(2), 205–214.



- Moulin-Frier, C., & Oudeyer, P. Y. (2013). Exploration strategies in developmental robotics: A unified probabilistic framework. In *2013 IEEE 3rd Joint International Conference on Development and Learning and Epigenetic Robotics, ICDL 2013 - Electronic Conference Proceedings*. IEEE.  
<https://doi.org/10.1109/DevLrn.2013.6652535>
- Nehmzow, U. (2006). Scientific methods in mobile robotics: Quantitative analysis of agent behaviour. In *Scientific Methods in Mobile Robotics: Quantitative Analysis of Agent Behaviour* (pp. 1–207). Springer-Verlag.  
<https://doi.org/10.1007/1-84628-260-8>
- Nighot, M. K., Patil, V. H., & Mani, G. S. (2012). Multi-robot hunting based on Swarm Intelligence. In *Proceedings of the 2012 12th International Conference on Hybrid Intelligent Systems, HIS 2012* (pp. 203–206). IEEE. <https://doi.org/10.1109/HIS.2012.6421334>
- Parsa, S., & Saadat, M. (2021). Human-robot collaboration disassembly planning for end-of-life product disassembly process. *Robotics and Computer-Integrated Manufacturing*, *71*, 102170.  
<https://doi.org/10.1016/j.rcim.2021.102170>
- Pontryagin, L. S. (1988). *Collected works*. Nauka, Moscow.
- Rikhsiev, B. B. (1989). The differential games with simple motions. *Tashkent: Fan*.
- Schaal, S., & Atkeson, C. G. (2010). Learning control in robotics. *IEEE Robotics and Automation Magazine*, *17*(2), 20–29. <https://doi.org/10.1109/MRA.2010.936957>
- Stone, P., & Veloso, M. (2000). Multiagent systems: a survey from a machine learning perspective. In *Autonomous Robots* (Vol. 8, Issue 3). Defense Technical Information Center. <https://doi.org/10.1023/A:1008942012299>
- Subbotin, A. I., & Chentsov, A. G. (1981). Optimization of guarantee in control problems. *Hauka. Moscow (in Rus-Sian)*, 288.
- Teslya, N., Smirnov, A., Ionov, A., & Kudrov, A. (2021). Multi-robot coalition formation for precision agriculture scenario based on gazebo simulator. In *Smart Innovation, Systems and Technologies* (Vol. 187, pp. 329–341). Springer Singapore. [https://doi.org/10.1007/978-981-15-5580-0\\_27](https://doi.org/10.1007/978-981-15-5580-0_27)
- Thrun, S. (2000). Probabilistic algorithms in robotics. *AI Magazine*, *21*(4), 93–109.
- Vajda, S. (1967). *Differential Games. A Mathematical Theory with Applications to Warfare and Pursuit, Control and Optimization*. By Rufus Isaacs. Pp. xxii, 384. 113s. 1965. (Wiley). *The Mathematical Gazette*, *51*(375), 80–81. <https://doi.org/10.2307/3613661>
- Varghese, B., & McKee, G. T. (2009). Modeling and simulating a mathematical tool for multi-robot pattern transformation. In *Proceedings - 2009 International Conference on Computer Modeling and Simulation, ICCMS 2009* (pp. 21–27). IEEE. <https://doi.org/10.1109/ICCMS.2009.40>