

# Bayesian Analysis of shape parameter of the Exponentiated Inverse Rayleigh distribution

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## Abstract

*The exponentiated inverse Rayleigh distribution (EIRD) is a generalization of inverse Rayleigh distribution introduced by Rao & Mbwambo (2019). Rao & Mbwambo (2019) investigated different methods of parameter estimation such as maximum likelihood method, percentile-based method, least squares method and weighted least squares method and compare their estimates using extensive numerical simulations. In this work, however, we explore Bayesian techniques to parameter estimation using symmetric and asymmetric loss function under informative and non-informative priors and compared the results with the best of what was obtained by Rao & Mbwambo (2019). The result shows that: Bayes estimator under the square error loss function assuming Jeffreys prior gave the same estimates with maximum likelihood estimators but slightly different and better when Bayesian estimator under square error loss function assuming gamma prior is considered which is a better alternative to the classical estimator.*

**Keywords:** Bayes Estimators; Jeffrey's Prior; Quadratic Loss Function; Precautionary Loss Function; Squared Error Loss Function; Monte Carlo simulation

## 1. Introduction

The Inverse Rayleigh distribution which was introduced by Trayer (1964) has been seen by many researchers a better and more flexible distribution than the earlier distribution ie Rayleigh distribution which was named after Lord Rayleigh and many research has been done in this area to further improve the work of Trayer (1964) and many distributions have been introduced such as the modified inverse Rayleigh distribution which was introduced by Muhammad (2014), the transmuted inverse Rayleigh distribution by Afaq Ahmad and Ahmed (2014) and lately the exponentiated inverse Rayleigh distribution by Rao and Mbwambo (2019) which we take a keen interest to.

The estimation of parameters of probability models is no new feat in statistical inferences and modelling as it's a necessary procedure to make relevant sense of situations where these models are applied, over the years researchers have gone extra miles to ensure better and more efficient methods of estimation of the parameter of this distributions are achieved, and in statistical estimation, there are clearly two sets of individuals with varying view of estimations some prefer the classical approach while others the Bayesians' approach. This classes of estimation has been applied in vast fields over the years especially in reliability studies and life data analysis, though the most widely used method for estimating distribution parameters has been the maximum likelihood estimation due to its very relevant property for providing the best linear unbiased estimate but other estimators have their own salient properties that make them relevant and in recent times, Bayesian approach of statistical estimation has aroused great consideration by many researchers among them is Yahaya and Dewu (2017) in their study estimated the scale parameter of log-logistic distribution under the assumption of informative priors wherein the Bayes estimates and posterior risks were derived under squared error loss function(SELF) and precautionary loss function(PLF), Aliyu and Yahaya (2016) acquired the estimates of the shape parameter of the generalized Rayleigh distribution using the squared error, Entropy and Precautionary loss functions assuming

non-informative priors and they compared their results with the maximum likelihood estimates (MLE) too. Naji and Rasheed (2019) studied Bayesian analysis involving the parameters estimation for the Gamma distribution taking into consideration the Generalized Weighted loss function, they assumed the Gamma prior and the Exponential prior for estimating the scale parameter and shape parameter. Method of the moment and Maximum likelihood estimators were the classical techniques considered and Lindleys approximation was used efficiently in the Bayesian estimation. And banking on results from the Monte Carlo simulations, these estimators were compared on the basis of the mean squared errors, Usta and Akdede (2019) studied the Bayesian analysis on the estimation of the two parameters of the reduced Kies distribution which is a special case of the Kies distribution. They took into consideration the independent gamma priors as the prior for the two unknown parameters, the Bayes estimators of the parameters were found with the aid of both the squared error and general entropy loss functions. Since the obtained posterior function could not be reduced logically to the familiar distributions, an amalgam Markov chain Monte Carlo (MCMC) method was implemented to compute the estimates of the two parameters using Bayesian statistics. The relative classical estimators of the two parameters which is the maximum likelihood estimators (MLEs) were acquired and the performances of the estimators were compared with the aid of a Monte Carlo simulation study. They also analyzed real-life data set to illustrate their work, Bayes estimators of the two parameters of the special Kies distribution ie the reduced Kies distribution (RKD) considering the SELF and GELF assuming the gamma priors were obtained by using the amalgam MCMC method, they computed the maximum likelihood estimates. And with the aid of a Monte Carlo simulation study, the comparison between the estimators was achieved. Salah (2018) conducted a study to evaluate the “maximum likelihood estimator, the Bayesian estimators and the approximate likelihood estimator” of a parameter for the Marshall-Olkin exponential distribution specifically its scale parameter under a progressive type-II censored sample. they derived and presented the “Bayes estimator, maximum likelihood estimator and approximate likelihood estimator” in simplified forms. they found out that the Bayesian estimator and the maximum likelihood estimator cannot be solved theoretically, therefore it was approached numerically to be able to solve it. they further made a comparison of the various methods of estimations that they had presented. To know the difference in performances among these estimators. The AMLE was used as a basis in obtaining the MLE and precise approximations for posterior moments and marginal densities method introduced by Tierney and Kadane (1986) was applied to get the numerical approximate solution of the Bayesian estimator.

Rao and Mbwambo (2019) obtained the Exponentiated Inverse Rayleigh Distribution with two parameters: a shape and a scale parameter and gave its properties their study considered the estimation of the parameters using “maximum likelihood estimators, percentile-based estimators, least squares estimators, and weighted least squares estimators” and compare those estimates using extensive numerical simulations. and concluded that the maximum likelihood estimates of the obtained distribution outst the performance of the other estimates obtained using the other methods of parametric estimations he considered from the classical perspective, so this paper seeks to estimate the shape parameter of this distribution using the Bayesian statistics when assuming both a non-informative and an informative prior and hence compare the results obtained with its corresponding maximum likelihood estimates.

## 2. Methodologies

### 2.1. The Exponentiated inverse Rayleigh distribution (EIRD)

The probability density function of the exponentiated inverse Rayleigh distribution is given as

$$f(x) = \frac{2\alpha\sigma^2}{x^3} e^{-\left(\frac{\sigma}{x}\right)^2} \left(1 - e^{-\left(\frac{\sigma}{x}\right)^2}\right)^{\alpha-1}; x \geq 0, \sigma, \alpha > 0 \quad (1)$$

and the cumulative density function is given as

$$1 - \left(1 - e^{-\left(\frac{\sigma}{x}\right)^2}\right)^\alpha \quad (2)$$

where  $\sigma$  is the scale parameter and  $\alpha$  is the shape parameter,  $X$  is the random variable.

This study will employ the following methods and techniques to estimate the shape parameter of the exponentiated inverse Rayleigh distribution (EIRD).

### 2.2. Maximum Likelihood Estimator

The likelihood function is given as

$$L(x|\alpha) \propto \alpha^n \prod_{i=1}^n \left(1 - e^{-\left(\frac{\sigma}{x}\right)^2}\right)^{\alpha-1} \quad (3)$$

The MLE is given as

$$\begin{aligned} \frac{d^2 \ln L(x|\alpha, \sigma)}{d\sigma} &\propto \frac{n}{\alpha} + \sum_{i=0}^n \left(1 - e^{-\left(\frac{\sigma}{x}\right)^2}\right) = 0 \\ \frac{n}{\alpha} &= \sum_{i=0}^n \left(1 - e^{-\left(\frac{\sigma}{x}\right)^2}\right) \\ n &= \alpha \sum_{i=0}^n \left(1 - e^{-\left(\frac{\sigma}{x}\right)^2}\right) \\ \alpha_{MLE} &= \frac{-n}{\sum_{i=0}^n \left(1 - e^{-\left(\frac{\sigma}{x}\right)^2}\right)} \end{aligned} \quad (4)$$

### 2.3. Bayesian Estimators

In this section, we obtain some Bayesian estimators for the shape parameter of the exponentiated inverse Rayleigh distribution EIRD, assuming the non-informative and informative prior under the square error loss function, precautionary loss function and quadratic loss function.

#### 2.4. Prior distribution

##### 2.4.1. Jeffrey prior

Jeffrey (1946) introduced a method for generating non-informative priors which were based on the principles of the Fishers information index or matrix denoted as  $I(\alpha)$  for vectors. let  $\pi_1(\alpha)$  denote the prior distribution of  $\alpha$  under Jeffreys prior.

$$\Rightarrow \pi_1(\alpha) = \sqrt{I(\alpha)} \quad (5)$$

Where  $I(\alpha)$  is the Fishers Information expressed as

$$\sqrt{I(\alpha)} = \sqrt{-E \left[ \frac{d^2 \ln L(x|\alpha, \sigma)}{d\sigma} \right]} \propto \sqrt{\frac{1}{\alpha^2}} = \frac{1}{\alpha} \quad (6)$$

##### 2.4.2. Gamma prior

The gamma prior is an informative prior that can sometimes be used as a conjugate prior and it is expressed as follows; Let  $\pi_2(\alpha)$  denote the prior distribution under Gamma

$$\Rightarrow \pi_2(\alpha) = \frac{b^a}{\Gamma(a)} \alpha^{a-1} e^{-ab} \quad (7)$$

## 2.5. Posterior distribution

The posterior distribution of alpha given the exponentiated inverse Rayleigh distribution EIRD is defined as the product of the prior distribution and the likelihood function expressed as

### 2.5.1. under the Jeffrey prior

$$P(\alpha_J|x) = \frac{\alpha^n \prod_{i=1}^n \left(1 - e^{-\left(\frac{\sigma}{x}\right)^2}\right)^{\alpha-1} \times \frac{1}{\alpha}}{g(x)}$$

where  $g(x)$  can be expressed as

$$g(x) = \int \alpha^n \prod_{i=1}^n \left(1 - e^{-\left(\frac{\sigma}{x}\right)^2}\right)^{\alpha-1} \times \frac{1}{\alpha} d\alpha$$

$$= \frac{\prod_{i=1}^n \left(1 - e^{-\left(\frac{\sigma}{x}\right)^2}\right)^{-1} \Gamma(n)}{\left(\sum \ln \left(1 - e^{-\left(\frac{\sigma}{x}\right)^2}\right)^{-1}\right)^n} \tag{8}$$

$$\Rightarrow P(\alpha_J|x) = \frac{\alpha^{n-1} \left(\sum \ln \left(1 - e^{-\left(\frac{\sigma}{x}\right)^2}\right)^{-1}\right)^n e^{-\alpha \sum \ln \left(1 - e^{-\left(\frac{\sigma}{x}\right)^2}\right)^{-1}}}{\Gamma(n)}$$

### 2.5.2. under the gamma prior

$$P(\alpha_G|x) = \frac{\alpha^n \prod_{i=1}^n \left(1 - e^{-\left(\frac{\sigma}{x}\right)^2}\right)^{\alpha-1} \times \frac{b^a}{\Gamma(a)} \alpha^{a-1} e^{-ab}}{g(x)}$$

where  $g(x)$  can be expressed as

$$g(x) = \int \alpha^n \prod_{i=1}^n \left(1 - e^{-\left(\frac{\sigma}{x}\right)^2}\right)^{\alpha-1} \times \frac{b^a}{\Gamma(a)} \alpha^{a-1} e^{-ab} d\alpha$$

$$= \frac{\prod_{i=1}^n \left(1 - e^{-\left(\frac{\sigma}{x}\right)^2}\right)^{-1} b^a \Gamma(n+a)}{\left(b + \sum \ln \left(1 - e^{-\left(\frac{\sigma}{x}\right)^2}\right)^{-1}\right)^{n+a} \Gamma(a)} \tag{9}$$

$$\Rightarrow P(\alpha_G|x) = \frac{\alpha^{n+a-1} \left(b + \sum \ln \left(1 - e^{-\left(\frac{\sigma}{x}\right)^2}\right)^{-1}\right)^{n+a} e^{-\alpha \left(b + \sum \ln \left(1 - e^{-\left(\frac{\sigma}{x}\right)^2}\right)^{-1}\right)}}{\Gamma(n+a)}$$

### 3. Loss function

#### 3.1. Squared error loss function

This is one of the universally used loss function which is defined by Legendre (1805) and Gauss (1810) as

$$\Rightarrow L(\hat{\alpha}, \alpha) = (\alpha - \hat{\alpha})^2 \quad (10)$$

where  $\hat{\alpha}$  is the Bayes estimator relative to SELF given as follows Since  $\hat{\alpha}$  gives the estimate that minimizes the risk  $R(\alpha ; \hat{\alpha})$  which is given as

$$\begin{aligned} \frac{dR(\theta, \hat{\alpha})}{d\alpha} &= \frac{d \int_{-\infty}^{\infty} (\alpha - \hat{\alpha})^2 \pi(\alpha | x) d\alpha}{d\alpha} = 0 \\ \Rightarrow \hat{\alpha} &= \int_{-\infty}^{\infty} \hat{\alpha} \pi(\alpha | x) d\alpha \equiv E(\alpha_{SELF|x}) \end{aligned} \quad (11)$$

and its posterior risk is expressed as

$$R_{SELF} = E(\alpha^2) - [E(\alpha)]^2 = E(\alpha^2|x) - [E(\alpha|x)]^2 \quad (12)$$

SELF is a symmetric loss function and it allots equal amount of losses to both overestimation and underestimation of any parameter of interest, which could be impractical in reality as consequences for overestimation and underestimation could be severe. The SELF is frequently used as it does not require extensive numerical computation.

#### 3.2. Precautionary loss function

This function by Norstrom (1996) is an asymmetric loss function which is mathematically expressed as

$$\Rightarrow L(\hat{\alpha}, \alpha) = \frac{(\hat{\alpha} - \alpha)^2}{\hat{\alpha}} \quad (13)$$

where  $\hat{\alpha}$  is the Bayes estimator relative to PLF given as follows Since  $\hat{\alpha}$  gives the estimate that minimizes the risk  $R(\alpha ; \hat{\alpha})$  which is given as

$$\begin{aligned} \frac{dR(\theta, \hat{\alpha})}{d\alpha} &= \frac{d \int_{-\infty}^{\infty} (\alpha - \hat{\alpha})^2 \pi(\alpha | x) d\alpha}{d\alpha} = 0 \\ \Rightarrow \hat{\alpha} &= \sqrt{E(\alpha^2)} \equiv [E(\alpha^2_{PLF})]^{1/2} \end{aligned} \quad (14)$$

and its posterior risk is expressed as

$$R_{PLF} = 2 \left[ \sqrt{E(\alpha^2)} - E(\alpha) \right] = 2[\alpha_{PLF} E(\alpha|x)] \quad (15)$$

This loss function is called an asymmetric loss function as it unlike the symmetric loss function only considers the loss of greater importance be it overestimation or underestimation, and is often preferred in reality.

#### 3.3. Quadratic loss function

Also, this is a universally used function due to it being mathematically more tractable than other loss functions sue to its variance property, this loss function like the former is also an asymmetric loss function. it is expressed as

$$\Rightarrow L(\hat{\alpha}, \alpha) = \left[ \frac{(\alpha - \hat{\alpha})}{\alpha} \right]^2 \quad (16)$$

which is given as

$$\frac{dR(\theta, \hat{\alpha})}{d\alpha} = \frac{d \int_{-\infty}^{\infty} \left[ \frac{(\alpha - \hat{\alpha})^2}{\alpha} \right] \pi(\alpha | x) d\alpha}{d\alpha} = 0$$

$$\Rightarrow \hat{\alpha}_{QLF} = \frac{\int_{-\infty}^{\infty} \alpha^{-1} \pi(\alpha | x) d\alpha}{\int_{-\infty}^{\infty} \alpha^{-2} \pi(\alpha | x) d\alpha} \equiv \frac{E(\alpha^{-1}|x)}{E(\alpha^{-2}|x)} \quad (17)$$

and its posterior risk is expressed as

$$R_{PLF} = 1 - \frac{[E(\alpha^{-1}|x)]^2}{E(\alpha^{-2}|x)} \quad (18)$$

from the information in this section, we can obtain the table 1

Table 1. Bayesian estimators and their posterior risks.

Loss function		Jeffrey prior	Gamma prior
SELF	Bayes estimator	$\frac{E(\alpha_{JSELF})}{n} = \frac{1}{\sum_{i=0}^n \ln \left( 1 - e^{-\left(\frac{\sigma}{\bar{x}}\right)^2} \right)^{-1}}$	$\frac{E(\alpha_{GSELF})}{n+a} = \frac{1}{b + \sum_{i=0}^n \ln \left( 1 - e^{-\left(\frac{\sigma}{\bar{x}}\right)^2} \right)^{-1}}$
	Posterior risk	$\frac{R_{JSELF}}{n} = \frac{1}{\left[ \sum_{i=0}^n \ln \left( 1 - e^{-\left(\frac{\sigma}{\bar{x}}\right)^2} \right)^{-1} \right]^2}$	$\frac{R_{GSELF}}{n+a} = \frac{1}{\left[ b + \sum_{i=0}^n \ln \left( 1 - e^{-\left(\frac{\sigma}{\bar{x}}\right)^2} \right)^{-1} \right]^2}$
PLF	Bayes estimator	$\frac{E(\alpha_{JPLF})}{\sqrt{(n+1)n}} = \frac{1}{\sum_{i=0}^n \ln \left( 1 - e^{-\left(\frac{\sigma}{\bar{x}}\right)^2} \right)^{-1}}$	$\frac{E(\alpha_{GPLF})}{\sqrt{(n+a)(n+a+1)}} = \frac{1}{b + \sum_{i=0}^n \ln \left( 1 - e^{-\left(\frac{\sigma}{\bar{x}}\right)^2} \right)^{-1}}$
	Posterior risk	$R_{JSELF} = 2 \left[ \frac{\sqrt{(n+1)n} - n}{\left[ \sum_{i=0}^n \ln \left( 1 - e^{-\left(\frac{\sigma}{\bar{x}}\right)^2} \right)^{-1} \right]} \right]$	$R_{GSELF} = 2 \left[ \frac{\sqrt{(n+a)(n+a+1)} - (n+a)}{\left[ b + \sum_{i=0}^n \ln \left( 1 - e^{-\left(\frac{\sigma}{\bar{x}}\right)^2} \right)^{-1} \right]} \right]$
QLF	Bayes estimator	$\frac{E(\alpha_{JQLF})}{n-2} = \frac{1}{\sum_{i=0}^n \ln \left( 1 - e^{-\left(\frac{\sigma}{\bar{x}}\right)^2} \right)^{-1}}$	$\frac{E(\alpha_{JQLF})}{n+a-2} = \frac{1}{b + \sum_{i=0}^n \ln \left( 1 - e^{-\left(\frac{\sigma}{\bar{x}}\right)^2} \right)^{-1}}$
	Posterior risk	$R_{JQLF} = \frac{1}{n-1}$	$R_{JQLF} = \frac{1}{n+a-1}$

#### 4. Simulation study

In this section, Monte Carlo simulation is employed to compare the performance of different estimates, for the estimates of the shape parameter of the exponentiated inverse Rayleigh distribution, Hyndman and Fan (1996) expressed the quantile function of distributions (say f(x)) as.

$$Q(u) = F^{-1}(u) \tag{19}$$

for  $0 < u < 1$  and  $F(x)$  the CDF of the Exponentiated inverse Rayleigh distribution (EIRD) equating the CDF to  $u$  we have the equation below

$$F(x) = 1 - \left(1 - e^{-\left(\frac{\sigma}{x}\right)^2}\right)^\alpha = u \tag{20}$$

Simplifying equation 19 above and making  $x$  the subject of the formula, we obtain:

$$x = \frac{\sigma}{\sqrt{-\ln(1 - (1 - u)^{\frac{1}{\alpha}})}} \tag{21}$$

The Maximum Likelihood and Bayes estimates and posterior risks was calculated with a rationally high amount of replications, samples with various sizes were considered to examine how the estimators derived performed with reference to their mean squared errors and biases. The performances of both classical and Bayesian estimators was evaluated using the following measures:

$$\text{Bias} = \left| \frac{1}{n} \sum_{i=0}^n (\hat{\alpha} - \alpha)^2 \right| \tag{22}$$

and

$$\text{Mean Square Error(MSE)} = \frac{1}{n} \sum_{i=0}^n (\hat{\alpha} - \alpha)^2 \tag{23}$$

and also the posterior risks, where  $\hat{\alpha}_i$  is the computed value of the parameter of interest from the  $i$ th simulated sample from the EIRD with shape parameter  $\alpha$  and scale parameter  $\sigma$  and also  $n$  is the number of Monte Carlo samples and the basis for comparison is that the estimator with the lowest bias MSE and posterior risk will be considered as the best. Simulations will consider when  $\alpha$  is less than, greater than and equal to  $\alpha$  and also at varying sample sizes

## 5. Result

Table 2. Estimators, their estimates, biases, mean square errors and posterior risks based on the sample sizes when  $\alpha = 0.5$  and  $\sigma = 1$ .

sample size (n)	Measure	MLE	JSELF	JPLF	JQLF	GSELF	GPLF	GQLF
10	Estimate	0.5508	0.5508	0.5776	0.4406	0.6991	0.6754	0.6059
	Bias	0.0508	0.0508	0.0776	0.0594	0.1991	0.1754	0.1059
	MSE	0.0400	0.0400	0.0472	0.0275	0.0797	0.0681	0.0413
	Posterior risk	NA	0.0348	0.0543	0.1111	0.0359	0.0462	0.0714
40	Estimate	0.5127	0.5127	0.5191	0.4871	0.5549	0.5487	0.5303
	Bias	0.0127	0.0127	0.0191	0.0129	0.0549	0.0487	0.0303
	MSE	0.0071	0.0071	0.0075	0.0065	0.0105	0.0097	0.0078

	Posterior risk	NA	0.0067	0.0127	0.0256	0.0070	0.0123	0.0227
60	Estimate	0.5083	0.5083	0.5125	0.4913	0.5368	0.5326	0.5203
	Bias	0.0083	0.0083	0.0125	0.0087	0.0368	0.0326	0.0203
	MSE	0.0045	0.0045	0.0047	0.0042	0.0060	0.0057	0.0048
	Posterior risk	NA	0.0044	0.0085	0.0169	0.0045	0.0082	0.0156
95	Estimate	0.5049	0.5049	0.5075	0.4942	0.5230	0.5204	0.5126
	Bias	0.0049	0.0049	0.0075	0.0058	0.0230	0.0204	0.0126
	MSE	0.0027	0.0027	0.0028	0.0026	0.0033	0.0032	0.0029
	Posterior risk	NA	0.0027	0.0053	0.0106	0.0028	0.0052	0.0101
195	Estimate	0.5023	0.5023	0.5036	0.4971	0.5112	0.5099	0.5061
	Bias	0.0023	0.0023	0.0036	0.0029	0.0112	0.0099	0.0061
	MSE	0.0013	0.0013	0.0013	0.0013	0.0014	0.0014	0.0013
	Posterior risk	NA	0.0013	0.0026	0.0052	0.0013	0.0026	0.0050

Table 3. Estimators, their estimates, biases, mean square errors and posterior risks based on the sample sizes when  $\alpha = 1$  and  $\sigma = 1$ .

sample size (n)	Measure	MLE	JSELF	JPLF	JQLF	GSELF	GPLF	GQLF
10	Estimate	1.1015	1.1015	1.1553	0.8812	1.2162	1.1749	1.0540
	Bias	0.1015	0.1015	0.1553	0.1188	0.2162	0.1749	0.0540
	MSE	0.1601	0.1601	0.1889	0.1100	0.1358	0.1137	0.0698
	Posterior risk	NA	0.1392	0.1086	0.1111	0.1061	0.0803	0.0714
40	Estimate	1.0254	1.0254	1.0382	0.9742	1.0694	1.0574	1.0218
	Bias	0.0254	0.0254	0.0382	0.0258	0.0694	0.0574	0.0218
	MSE	0.0285	0.0300	0.0300	0.0258	0.0307	0.0286	0.0241
	Posterior risk	NA	0.0255	0.0255	0.0256	0.0260	0.0236	0.0227
60	Estimate	1.0166	1.0166	1.0250	0.9827	1.0472	1.0391	1.0150
	Bias	0.0166	0.0166	0.0250	0.0173	0.0472	0.0391	0.0150
	MSE	0.0180	0.0180	0.0186	0.0168	0.0191	0.0182	0.0161
	Posterior risk	NA	0.0176	0.0169	0.0169	0.0172	0.0161	0.0156
95	Estimate	1.0098	1.0098	1.0151	0.9885	1.0297	1.0246	1.0091
	Bias	0.0098	0.0098	0.0151	0.0115	0.0297	0.0246	0.0091
	MSE	0.0109	0.0109	0.0112	0.0105	0.0115	0.0111	0.0102
	Posterior risk	NA	0.0109	0.0106	0.0106	0.0107	0.0103	0.0101
195	Estimate	1.0046	1.0046	1.0072	0.9943	1.0146	1.0121	1.0044
	Bias	0.0046	0.0046	0.0072	0.0057	0.0146	0.0121	0.0044
	MSE	0.0052	0.0052	0.0053	0.0051	0.0053	0.0052	0.0050
	Posterior	NA	0.0052	0.0051	0.0052	0.0052	0.0051	0.0050



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Table 4. Estimators, their estimates, biases, mean square errors and posterior risks based on the sample sizes when  $\alpha = 1.5$  and  $\sigma = 1$ .

sample size (n)	Measure	MLE	JSELF	JPLF	JQLF	GSELF	GPLF	GQLF
10	Estimate	1.6523	1.6523	1.7329	1.3218	1.6172	1.5623	1.4016
	Bias	0.1523	0.1523	0.2329	0.1782	0.1172	0.0623	0.0984
	MSE	0.3602	0.3602	0.4250	0.2474	0.1358	0.1178	0.1014
	Posterior risk	NA	0.3132	0.1628	0.1111	0.1847	0.1067	0.0714
40	Estimate	1.5382	1.5382	1.5573	1.4613	1.5477	1.5304	1.4789
	Bias	0.0382	0.0382	0.0573	0.0387	0.0477	0.0304	0.0211
	MSE	0.0642	0.0642	0.0676	0.0581	0.0525	0.0501	0.0463
	Posterior risk	NA	0.0607	0.0382	0.0256	0.0543	0.0342	0.0227
60	Estimate	1.5249	1.5249	1.5375	1.4740	1.5332	1.5214	1.4860
	Bias	0.0249	0.0249	0.0375	0.0260	0.0332	0.0214	0.0140
	MSE	0.0404	0.0404	0.0419	0.0379	0.0356	0.0344	0.0326
	Posterior risk	NA	0.0396	0.0254	0.0169	0.0368	0.0235	0.0156
95	Estimate	1.5146	1.5146	1.5226	1.4827	1.5209	1.5132	1.4904
	Bias	0.0146	0.0146	0.0226	0.0173	0.0209	0.0132	0.0096
	MSE	0.0246	0.0246	0.0252	0.0237	0.0228	0.0223	0.0216
	Posterior risk	NA	0.0244	0.0159	0.0106	0.0234	0.0152	0.0101
195	Estimate	1.5069	1.5069	1.5107	1.4914	1.5103	1.5066	1.4952
	Bias	0.0069	0.0069	0.0107	0.0086	0.0103	0.0066	0.0048
	MSE	0.0117	0.0117	0.0118	0.0115	0.0113	0.0112	0.0110
	Posterior risk	NA	0.0117	0.0077	0.0052	0.0114	0.0075	0.0050

Table 5. Estimators, their estimates, biases, mean square errors and posterior risks based on the sample sizes when  $\alpha = 2.0$  and  $\sigma = 1$ .

sample size (n)	Measure	MLE	JSELF	JPLF	JQLF	GSELF	GPLF	GQLF
10	Estimate	2.2030	2.2030	2.3106	1.7624	1.9385	1.8728	1.6800
	Bias	0.2030	0.2030	0.3106	0.2376	0.0615	0.1272	0.3200
	MSE	0.6404	0.6404	0.7555	0.4399	0.1446	0.1476	0.2082
	Posterior risk	NA	0.5569	0.2171	0.1111	0.2627	0.1278	0.0714
40	Estimate	2.0509	2.0509	2.0764	1.9483	1.9936	1.9713	1.9050

	Bias	0.0509	0.0509	0.0764	0.0517	0.0064	0.0287	0.0950
	MSE	0.1141	0.1141	0.1201	0.1033	0.0777	0.0767	0.0799
	Posterior risk	NA	0.1079	0.0509	0.0256	0.0900	0.0440	0.0227
60	Estimate	2.0332	2.0332	2.0500	1.9654	1.9965	1.9811	1.9351
	Bias	0.0332	0.0332	0.0500	0.0346	0.0035	0.0189	0.0649
	MSE	0.0718	0.0718	0.0744	0.0673	0.0557	0.0552	0.0565
	Posterior risk	NA	0.0703	0.0338	0.0169	0.0624	0.0306	0.0156
95	Estimate	2.0195	2.0195	2.0301	1.9770	1.9972	1.9871	1.9572
	Bias	0.0195	0.0195	0.0301	0.0230	0.0028	0.0129	0.0428
	MSE	0.0438	0.0438	0.0448	0.0421	0.0374	0.0372	0.0377
	Posterior risk	NA	0.0435	0.0212	0.0106	0.0403	0.0199	0.0101
195	Estimate	2.0092	2.0092	2.0143	1.9886	1.9986	1.9936	1.9786
	Bias	0.0092	0.0092	0.0143	0.0114	0.0014	0.0064	0.0214
	MSE	0.0208	0.0208	0.0210	0.0204	0.0193	0.0192	0.0193
	Posterior risk	NA	0.0208	0.0103	0.0052	0.0200	0.0100	0.0050

## 1. Conclusions

The shape parameter was estimated using the squared error, the quadratic and the precautionary loss function assuming Jeffrey and gamma priors. when the Jeffreys prior was considered, the estimates under the SELF ie JSELF had the best estimates in terms of bias but the estimate under the QLF ie JQLF was best when the mean square error MSE and posterior risk. Similarly, when the gamma prior was considered, the estimates under the three loss function competed significantly across altered sample sizes and various values that the shape parameter could take but the estimates under the GQLF seemed to be more consistent. In the overall assessment the estimator under the Jeffreys prior when the square error loss function was used constantly gave the best estimates in terms of bias but the estimator under the gamma prior when the square error loss function was used constantly gave the best estimates when the means square errors MSE are considered and the one under the quadratic loss function consistently gave the best estimates in terms of the posterior risk when considered, all these conclusions hold irrespective of the sample size. When compared to the best classical estimator of this parameter the Bayes estimator obtained via the SELF assuming the Jeffreys prior JSELF gave exactly the same estimates with that of the maximum likelihood estimators as its formulas are equal to each other, and from this, we can say due to the tendency of high mean square error from the classical best estimator ie the maximum likelihood estimator the Bayesian estimator under square error loss function assuming gamma prior can be considered as a better alternative to the classical estimator.

## Abbreviations

The following abbreviations are used in this manuscript

JSELF-square error loss function assuming Jeffrey's prior

GSELF-square error loss function assuming gamma's prior

JQLF-quadratic loss function assuming Jeffrey's prior



GQLF-quadratic loss function assuming gamma's prior  
JPLF-precautionary loss function assuming Jeffrey's prior  
GPLF-precautionary loss function assuming gamma's prior  
MSE-mean square error

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