



Comparing the Performance of ARIMA and ARFURIMA Model in Forecasting Nigeria Stock Price Index

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Abstract

Financial indices are known to exhibit an upward or downward trend and are classified as a non-stationary time series. Therefore, many stationary tests are used to investigate and confirmed the presence of a unit root in this type of series. The first differenced transformation method using ($d=1$) is used to obtain a stationary component of this series and the Autoregressive Integral Moving Average (ARIMA) model estimated. Also, another dwelled and unwanted component in the similar financial time series is a long memory and is determined by using the autocorrelation function, long memory tests and fractional ARIMA could be the appropriate model to study the series. In this paper, a monthly Nigeria stock index representing thirty (30) stock price indices (N30SPI) between the 29th January 2010 and 31st December 2018 were analyzed using the ARIMA and Autoregressive Fractional Unit Root Integral Moving Average (ARFURIMA) model. The Kwiatkowski-Phillips-Schmidt-Shin (KPSS) and Geweke and Porter-Hudak (GPH) test results show that the stock index has the unit root and exhibit interminable long memory respectively with $d=1.245$ which is in the interval of $1 < d < 2$. Applying differencing, results show the fractional unit differencing methods eliminates the unwanted interminable long memory better than the first differencing. Furthermore, several ARIMA and ARFURIMA models were identified and estimated. Results indicate ARFURIMA is the best model in terms of fit, serial correlation analysis and accuracy measures. The in-sample forecasts show the adequacy of the ARFURIMA models. Consequently, the out-sample forecasts result show decreasing stock prices indicating that the ARFURIMA is the appropriate model for modelling and forecasting the monthly N30SPI data. This is because the forecast decreasing trend captured by the ARFURIMA model indicate the difficult situation currently witnessed in the Nigerian economy and other economies due to Corona Virus 2019 (COVID-19) pandemic impact. Finally, it is recommended that when the time series is known to have d , in the interval of $1 < d < 2$ applying fractional unit root differencing and estimating ARFURIMA model is appropriate. It helps in achieving reliable forecasts for making future inform-decision.

Keywords: Interminable long memory, GPH test, Fractional unit root differencing, ARFURIMA model and monthly stock price index.

1. Introduction

Some of the characteristics of financial data are non-stationarity and long memory. A time series is said to be non-stationary when there is evidence of trend (upward or downward) in the time plot. Financial data, for example, a stock index is non-stationary when Augmented Dickey-Fuller (ADF) or any other stationary tests are used to show that unit-roots are present. Dickey and Fuller (1981), Phillips and Perron (1988) and Kwiatkowski-Phillips-Schmidt-Shin (KPSS) known as KPSS (1992) were few among early studies on this unit-roots property. However, long memory occurs when autocorrelations in time series are, positive, large and occurs for many lags (Ho et al. (2013)). Long memory, also called long-range dependence, is a statistical property that may arise in the financial index. It was first discovered in geophysical data by Hurst (1951). According to Qu (2011), if the spectral density of a scalar K is proportional to K^{-2d} as K tend to zero, the



process is said to have long memory with d as the long memory parameter and also called as the degree of fractional differencing. Other explanations of this long memory both in time and frequency domain can be found in Hosking (1981), Geweke and Porter-Hudak (1983), Bai and Perron (2003) and Yakubu and Jibrin (2021).

2. Related Works

Recent works had used time series models to study the financial series. Analyzing stock index, the trend or nonstationary had been seen, unit root characteristics tested and Autoregressive Integral Moving Average (ARIMA) models estimated (see Abdullahi and Bakari (2014), Uzuke et al., (2016) and Eke (2019)). Similarly, the ARIMA is compared with the Autoregressive Fractional Integral Moving Average (ARFIMA) model when the degree of fractional differencing, d , had been estimated to be in the interval $0 < d < 1$. For example, Erfani and Samimi (2009) studied the daily Tehran Stock Price Index (TSPI) using the modified R/S and Hurst exponent methods. They found evidence of long memory in TSPI by applying Peters (1991) approach for estimating the fractional differencing parameter. They estimated the fractional differencing value as equal to 0.4767. The comparison of stationary ARIMA and ARFIMA models in their study revealed that the ARFIMA (2, 0.4767, 18) is the best model to study the daily TSPI. Also, in-sample and long-range out-of-sample forecast further supported the choices of the long memory models to study the TSPI data. Onali and Goddard (2011) investigated long memory in the returns of the stock index of eight European countries and the United State (US) by using the Rescaled Range Analysis. They found a high degree of long memory for the stock prices of Italy and the Czech Republic contrary to the claim that the market is efficient. However, the remaining six European markets showed no evidence or a very low long memory which was supporting the arguments that stock markets are efficient. Núñez et al. (2017) investigate the occurrence of long-range dependence by considering stocks of nineteen prominent world markets. The study was carried out to ascertain if the duration of data, interval of data or the nature of the available long memory models for analysis can affect the results. Parametric and nonparametric methods for estimating long memory parameters were used in addition to the ARFIMA models. Results indicated that there is a long memory effect in the South Korea and China index despite the type of models, the interval of data collected and the size(duration) of data used. However, the findings also showed that some markets stock index used in the study exhibits short memory behaviour.

The study of Ahmad (2013) investigates the dependence strength of the Kuala Lumpur Composite Index (KLCI) future contract and spot prices by using fractional integration and ARFIMA methods. His results also show the presence of long memory in the series. Also, the findings confirmed that it is possible to use previous prices to forecast both the future price values and dependency observed in the KLCI index. This is in line with Hyndman and Anathasopulos (2013) on obtaining or realizing a similar forecast behaviour that emulates the historical series used in a study. Furthermore, the paper of Gil-Alana et al. (2018) investigates long memory in the daily Baltic stock market indices by using fractional integration methods. Fractional differencing, d , was estimated to be in the interval $1 < d < 2$ only without estimating the long-memory model. They investigate the presence of structural breaks based on the range of data used and associates the two breaks to bull and bear market phases. They found evidence of long memory and estimate the fractional differencing values to be greater than one in each of the overall and sub-sample data. Furthermore, the volatility analysis indicates a high degree of long memory in the bear markets compared to the bull markets. Overall, they conclude that both the historical and returns of the Baltic indices are generated by a long memory process. For stock price index evaluation and modelling see Suleiman (2016) and Rahman and Jibrin (2019). As trend and seasonal variation are adjusted or removed by using transformation methods, the long memory is another emerging variation that is important to be eliminated from any economic and financial data that exhibits dependency. There are exist transformation filters or methods that were developed for obtaining the stationary component of the non-stationary financial data.

In the literature, first differencing, fractional differencing and fractional unit root differencing are some of the procedures used for eliminating trend and noise signal as used by, Emenike (2008), Granger and Newbold (2014) and Jibrin and Rahman (2019) respectively. Therefore, it is pertinent to note that for a non-stationary series, eliminating or reducing excess noise signal is a significant stage of every modelling and forecasting of time series.

When time-series data or in particular financial indices exhibit trend and non-stationarity, many researchers are tempted to apply the Box and Jenkins (1976) methods. This is specifically when the series are found to have unit-roots. The differencing parameter, d , is assumed to be unity ($d=1$), first differencing is carried out and the perception is to eliminate unwanted noise signals or variabilities to be stabilized. Subsequently, it was suggested the suitable model for studying any financial indices with these time series properties is the ARIMA (see Mathew et al., (2013), Adebisi et al. (2014), Jibrin (2015), Olayiwola et al., (2016), Ekong and Ebong (2017), Nyoni (2018), Okorie et al., (2019) and Musa and Joshua (2020)). However, Wei (2006), Nau (2014) and Rahman and Jibrin (2018) discussed the problems of under-differencing as it has bad effects on differenced series and models identification and specification. Another fact is from the works of Dalla (2015) and Sabzikar et al. (2019), they have estimated the degree of fractional difference value in the interval of $1 < d < 2$. Given these, there is still a need to apply the fractional filter of Rahman and Jibrin (2019) and Autoregressive Fractional Unit Root Integral Moving Average (ARFURIMA) model of Jibrin (2019). The fractional filter is suitable for eliminating unwanted interminable long memory component of a data that has memory parameter in the interval of $1 < d < 2$. The stationary series is obtained due to the fractional unit root differencing.

In this paper, a recent method that is capable of modelling and forecasting interminable long memory time series based on the ARFURIMA method is used. The method is compared with the existing ARIMA model and studies the monthly Nigeria stock exchange index. It's an index made up of the 30 biggest stocks by market capitalization. The rest of the paper is organized as follows; Section 2 gives materials and methodologies. Section 3 presents the numerical analysis and the main results of this paper. Section 4 presents the discussion of results while the conclusion is presented in Section 5.

2.1 Theoretical Review

Data used and the methods for testing interminable long memory are discussed in this section.

2.1.1 Data

Data for this study are obtained from the Morgan Stanley Capital International (MSCI) data stream and there are 108 monthly (12 months) Nigeria 30 Stock Price Index (N30SPI) between the 29th January 2010 and 31st December 2018.

2.1.2 The Non-stationary Models

Yule (1924) is the first work on Autoregressive (AR) and is a model in which one uses the statistical properties of the past behaviour of a variable to predict its behaviour in the future.

Consider the stationary series X_t with mean function $\mu_t = \mu$ is a constant, provided $E(|X_t|) < \infty$. Likewise, the variance function $\sigma_t^2 = \sigma^2$ for all t is also a constant provided $E(|X_t^2|) < \infty$ and hence, autocovariance $\gamma(t_1, t_2) = \gamma_k$, fo $t_1 = t - k$ and $t_2 = t$ (see Wei, 2006 pg. 7). Then, the general representation of an AR model of order p , AR(p) is written as:

$$X_t = \varphi_1 X_{t-1} + \varphi_2 X_{t-2} + \dots + \varphi_p X_{t-p} + \varepsilon_t, \quad (1)$$

where the term ε_t is the source of randomness and is called white noise. The $\varphi_1, \varphi_2, \dots, \varphi_p$ are unknown parameters relating X_t to X_{t-1}, \dots, X_{t-p} and must be estimated from sample data.

A process $\{\varepsilon_t\}$ is said to be a white noise process if it is a sequence of uncorrelated random variables with constant mean $E(\varepsilon_t) = \mu_\varepsilon$, usually assumed to be 0, constant variance $Var(\varepsilon_t) = \sigma_\varepsilon^2$ and $\gamma_k = Cov(\varepsilon_t, \varepsilon_{t+k}) = 0$ for all $k \neq 0$ (see Wei, 2006 pg. 15). Slutsky (1927) introduced the Moving Average (MA) model and the notation MA(q) also refers to the moving average model of order q given by:

$$X_t = \theta_1 \varepsilon_{t-1} + \dots + \theta_q \varepsilon_{t-q} + \varepsilon_t, \quad (2)$$

where $\theta_1, \dots, \theta_q$ are the parameters of the model, and $\varepsilon_t, \varepsilon_{t-1}, \dots, \varepsilon_{t-q}$ are white noise error terms.

The general Autoregressive Moving Average (ARMA) model was described by Whittle (1956) and it was popularized in the book of Box and Jenkins (1976). It can be seen from above that the AR model includes lagged terms on the time-series itself and that the MA model includes lagged terms on the noise or residuals. By including both types of lagged terms, we arrive at the ARMA model. Therefore ARMA(p, q), where p is the autoregressive order and q the moving-average order can generally be represented as:

$$\varphi(L)X_t = \theta(L)\varepsilon_t, \quad (3)$$

where $\varphi(L) = 1 - \varphi_1L - \varphi_2L^2 - \dots - \varphi_pL^p$ and $\theta(L) = 1 - \theta_1L - \theta_2L^2 - \dots - \theta_qL^q$. It is well known that for (3) to be stationary and invertible, the zeros of $\varphi(L)$ and $\theta(L)$ must be outside the unit circle respectively.

Most real-life time series are nonstationary. For such time series, Box and Jenkins (1976) proposed that differencing up to an order d could render it stationary. Therefore, ARIMA is the appropriate method to describe this type of time series.

Consider nonstationary time series Y_t and suppose the stationary d^{th} order difference of Y_t is denoted by $\Delta^d Y_t$ such that $\Delta^d Y_t = X_t$ where $\Delta^d = (1 - L)^d$. Putting Δ^d in (3) yields an ARIMA model of order p, d and q , denoted by ARIMA(p, d, q):

$$\varphi(L)\Delta^d Y_t = \theta(L)\varepsilon_t. \quad (4)$$

2.1.3 The Long Memory Models

The ARFIMA like ARIMA models has three parameters: p, d , and q . The parameter corresponding to the number of lags involved in the autoregressive portion of the series is p . Meanwhile, the parameter for the moving average lags is q . If the series is fractionally integrated, with d takes a value in the interval of $0 < d < 1$ then, the model is referred to as an ARFIMA model.

Now, consider $\{Y_t\}, t = 1, \dots, T$ is a nonstationary process with time-varying mean and variance. Then Y_t is said to be fractional integral process if

$$(1 - L)^d Y_t = \varepsilon_t, \quad (5)$$

where this has the interpretation as follows:

$$Y_t - dY_{t-1} + \frac{d(d-1)}{2!}Y_{t-2} - \dots = \varepsilon_t. \quad (6)$$

Here, L is the backward shift operator, ε_t is a white noise process and d is the long memory parameter such that $0 < d < 1$. The general form of an ARFIMA model of Granger and Joyeux (1980) and Hosking (1981) is given by:

$$\varphi(L)(1 - L)^d Y_t = \theta(L)\varepsilon_t, 0 < d < 1. \quad (7)$$

The simplest autoregressive fractionally integrated model, ARFIMA(0, d , 0), is, in standard notation as shown in (5). An ARFIMA model shares the same form of representation as to the ARIMA(p, d, q) process, specifically:

$$(1 - \sum_{i=1}^p \varphi_i L^i)(1 - L)^d Y_t = (1 - \sum_{j=1}^q \theta_j L^j)\varepsilon_t, \quad (8)$$

where the parameter d is a non-integer value, Y_t is the dependent variable at time t , ε_t is distributed normally with mean 0 and variance σ^2 , and $\sum_{i=1}^p \varphi_i L^i$ and $\sum_{j=1}^q \theta_j L^j$ represent AR and MA polynomials of order L , respectively.

In contrast to the ordinary ARIMA process, the "difference parameter", d , is allowed to take non-integer values. In a fractional model, the power is allowed to be fractional, with the meaning of the term identified using the following formal binomial series expansion

$$(1 - L)^d = \sum_{k=0}^{\infty} \binom{d}{k} (-L)^k = 1 - dL + \frac{d(d-1)}{2!} L^2 - \dots \quad (9)$$

2.1.4 The Interminable Long Memory Models

A time series that exhibits a very strong decaying (more than usual) ACF, with perfect and strong positive autocorrelations, with a degree of fractional difference values in the interval of $1 < d < 2$ and whose future behaviour can be perfectly predicted from its record can be said to be generated by interminable long memory process (Rahman and Jibrin, (2019)).

Given this, Jibrin (2019) introduced the ARFURIMA model to study time series with $1 < d < 2$. The general form of an ARFURIMA (p,d,q) model is given by:

$$\varphi(L)\{(1 - L)(1 - d^*(1 + L))\}Y_t = \theta(L)\varepsilon_t, \quad (10)$$

where $\varphi(L)$ and $\theta(L)$ are defined similarly to the stationary and invertible conditions in (3). L is the backward shift operator, ε_t is a white noise process and $\{(1 - L)(1 - d^*(1 + L))\}$ is the interminable long memory fractional filter. The filter fractionally differences a process that exhibits a very slow decaying ACF. Here, $d^* = d - 1$ such that, $0 < d^* < 1$ and $1 < d < 2$ and both d^* and d is a long memory and interminable long memory parameters respectively. The identification of the ARFURIMA(p,d,q) model follows the Box and Jenkins approach. See Rahman and Jibrin (2019) and Jibrin (2019) for the detail derivation of fractional filter, ARFURIMA model and asymptotic properties of (10).

Reference to the ARIMA model estimation as discussed by Yakubu and Jibrin (2021), the ARFURIMA model can be estimated using similar procedure as follows:

- 1) Estimate the fractional difference order, d , by GPH method.
- 2) Decompose or filter the interminable long memory series by using the interminable long memory fractional filter in (10).
- 3) Consider the Box and Jenkins (1976) approach to estimate the parameters of the ARFURIMA(p,d,q).
- 4) Select the best candidates of ARFURIMA model(s) according to the least AIC values.
- 5) Carry out the goodness-of-fits tests among the candidates of the ARFURIMA models.
- 6) Perform serial correlation test on the candidate models' residuals.
- 7) Carry out the forecast and validate the models forecast by comparing the accuracy measures.

2.1.5 The Kwiatkowski Phillips Schmidt and Shin Tests

For a non-stationary and long memory modelling, it is important to test and estimate the degree of interminable long memory before modelling and test the presence of heteroscedasticity after model estimation (Jibrin, 2015). In this study, we consider the KPSS for unit root test. The Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test has been proposed by Kwiatkowski et. al. (1992) with the null hypothesis that the data generating process is stationary are tested against a unit root. If the data is without a linear trend, then the test assumes

$$x_t = y_t + z_t, \quad (11)$$

where $y_t = \alpha_1 y_{t-1} + \dots + \alpha_p y_{t-p} + \varepsilon_t$, $\varepsilon_t \sim i.i.d(0, \sigma_\varepsilon^2)$ and z_t is a stationary process.

The test statistic is given by:

$$t - statistic(t_k) = \frac{1}{T^2} \sum_{t=1}^T \frac{s_t^2}{\hat{\sigma}_\infty^2}, \quad (12)$$

where $s_t = \sum_{j=1}^t \hat{w}_j$ with $\hat{w}_t = y_t - y$ and $\hat{\sigma}_\infty^2$ is an estimator of the long-run variance of the process z_t .

2.1.6 Geweke and Porter-Hudak Estimator

The Geweke and Porter-Hudak (GPH) for the test and estimation of the degree of interminable long memory. Given y_t , a fractionally integrated series, a semi-parametric method for the estimation of d denoted by d_{GPH} is:

$$\hat{d}_{GPH} \rightarrow N\left(d, \frac{\pi^2}{6 \sum_{i=1}^m (y_i - \bar{y})^2}\right), \quad (13)$$

where N is the sample size, \bar{y} is the mean of y_t and π is constant. See GPH (1983) for the derivation of equation (13).

2.1.6 ARCH-LM Test

Before estimating a full ARFURIMA model for a financial time series, it is a good practice to test for the adequacy of the model. The ARCH-Lagrange Multiplier (ARCH-LM) test based on the McLeod and Li (1983) determines if there is a significant dependence in the residuals obtained from the fit. The Ljung-Box (1978) statistic was used to determine whether the residuals are serially correlated or not and is given by:

$$Q = N(N + 2) \sum_{i=1}^N \frac{\rho_i}{(N-i)}, \quad (14)$$

where the Q statistic has an asymptotic distribution χ^2 with n degrees of freedom if square residuals are uncorrelated. N is the number of observation and ρ_i is the sample correlation coefficient between squared residuals $\hat{\varepsilon}_t^2$ and $\hat{\varepsilon}_{t-1}^2$. The null hypothesis of square residuals is the $\{\hat{\varepsilon}_t^2\}$ are uncorrelated.

2.1.7 Jarque Bera Test

Jarque and Bera (1987) have proposed a test for non-normality based on third and fourth moment known as skewness and kurtosis of a distribution respectively. The test checks the pair of hypothesis: $H_0: E(\varepsilon_t^2)^3 = 0$ and $E(\varepsilon_t^2)^4 = 3$ (the distribution is symmetry and therefore normal) vs $H_1: E(\varepsilon_t^2)^3 \neq 0$ and $E(\varepsilon_t^2)^4 \neq 3$ (the distribution is asymmetric and therefore non-normal). The test statistic is given as:

$$JB = \frac{T}{6} [T^{-1} \sum_{t=1}^T (\hat{\varepsilon}_t^2)^3]^T + \frac{T}{24} [T^{-1} \sum_{t=1}^T (\hat{\varepsilon}_t^2)^4 - 3]^T. \quad (15)$$

The test statistic has an asymptotic χ^2 distribution with 2 degrees of freedom if the null hypothesis is true. The null hypothesis is rejected if the p-value is less than significant level α .

2.1.8 Mean Error

Assessing the accuracy of forecasts obtained from a time series model is important but a difficult task. To handle this vital stage of modelling, this section presents two measures of forecasting performance. These are Mean Error (ME) and Root Mean Square Error (RMSE).

The ME is defined as the measure of the accuracy of forecasts. The ME of any proposed forecasting model ought to be very small. The deviations in forecasting errors explain the relationship between historical observations and the fitted values. The following equation is used in estimating the ME.

$$ME = \frac{1}{n} \sum_{t=1}^n \varepsilon_t, \quad (16)$$

where ε_t is the random error term. It is obtained using the relation $\varepsilon_t = Y_t - \hat{Y}_t$. Y_t is the output, \hat{Y}_t is the forecasts output and n is the sample size.

2.1.9 Root Mean Square Error

The RMSE is defined as the estimate of deviation of errors in forecasting. It is calculated by obtaining the square root of the difference among predicted and historical observations that are both squared and averaged over the sample. A smaller RMSE indicates a better model estimate. The RMSE expressed below in (17) is used to obtain the RMSE.

$$RMSE = \sqrt{\frac{1}{n} \sum_{t=1}^n (Y_t - \hat{Y}_t)^2}. \quad (17)$$

2.1.10 Forecast Accuracy Test

Diebold and Mariano (1995) introduced a test of forecast accuracy for two sets of forecasts. Diebold and Mariano's test statistic for the null hypothesis of equal forecast accuracy is:

$$DM = [\hat{V}(\bar{g})]^{-\frac{1}{2}} \bar{g}, \quad (18)$$

where DM has an asymptotic standard normal distribution under the null hypothesis.

2.1.11 Information Criteria

The Akaike Information Criterion (AIC) is a measure of the relative goodness of fit of a statistical model as well as the order of the model. Akaike (1974) suggests measuring the goodness of fit for some particular model by balancing the error of the fit against the number of parameters in the model. The formula for the AIC is:

$$AIC = 2w - 2 \ln(L), \quad (19)$$

where w is the number of parameters in the statistical model, and L is the maximized value of the likelihood function for the estimated model.

3. Results

This section provides the summary statistics, interminable long memory tests and time series modelling of M3ONSPI.

Figure 1 is the plot and ACF for N30SPI. The time series plot of the Nigeria stock exchange index as shown in Figure 1(a) exhibit an upward trend and this is due to an average increase in the stock index value. Also, the stock index appreciates and depreciate over time such as in 2012-2014 (second quarter) and 2014 (third quarter)-2017 respectively. Furthermore, the ACF in Figure 1(b) shows a very slow decay in long-term with positive autocorrelations which are evidence of long memory process. Given this, there exist long memory in the understudied series and it can be described as an interminable long memory. Series that possesses or displayed such time-series properties (interminable long memory) is presumed to have fractional unit root differencing value in the range, $1 < d < 2$. Having said this, the stock index for the 30 biggest stocks by market capitalization is generated by an interminable long memory process. Therefore, on the average, the series is not stationary.

The ACF and time series plots for both Fractional Unit Root Differenced (FURD) and First Differenced (FD) monthly N30SPI are shown in Figure 2. After lag 0, the autocorrelations of FURD series as shown in Figure 2(a) are smaller and within the confidence limit indicating stationarity. However, the FD series in Figure 2(b) is non-stationary due to spike at lag 1, 11 and 12. This indicates the weaknesses of FD approach when the series is known to have fractional differencing value in the interval of $1 < d < 2$. Similar results of stationarity and non-stationarity are obtained in Figure 2(c) and (d) respectively. Therefore, the plots in Figure 2 (a) and (c) shows that the series is stationary after fractional unit root differencing is conducted.

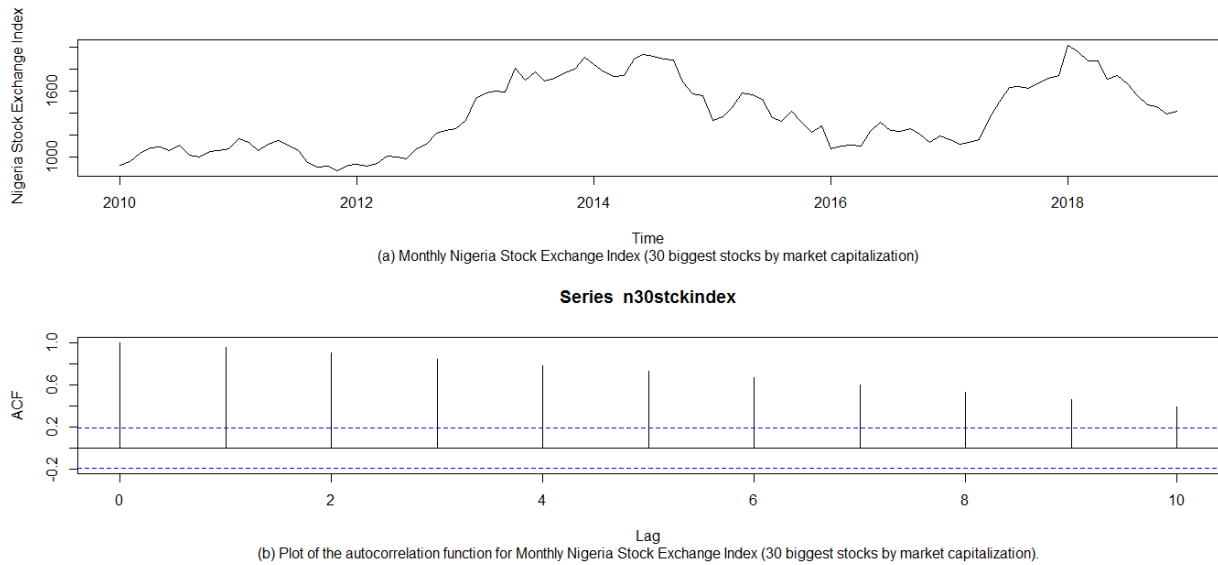


Figure 1: Time Series Plot and ACF for Monthly Nigeria Stock Exchange Index (30 biggest stocks by market capitalization).

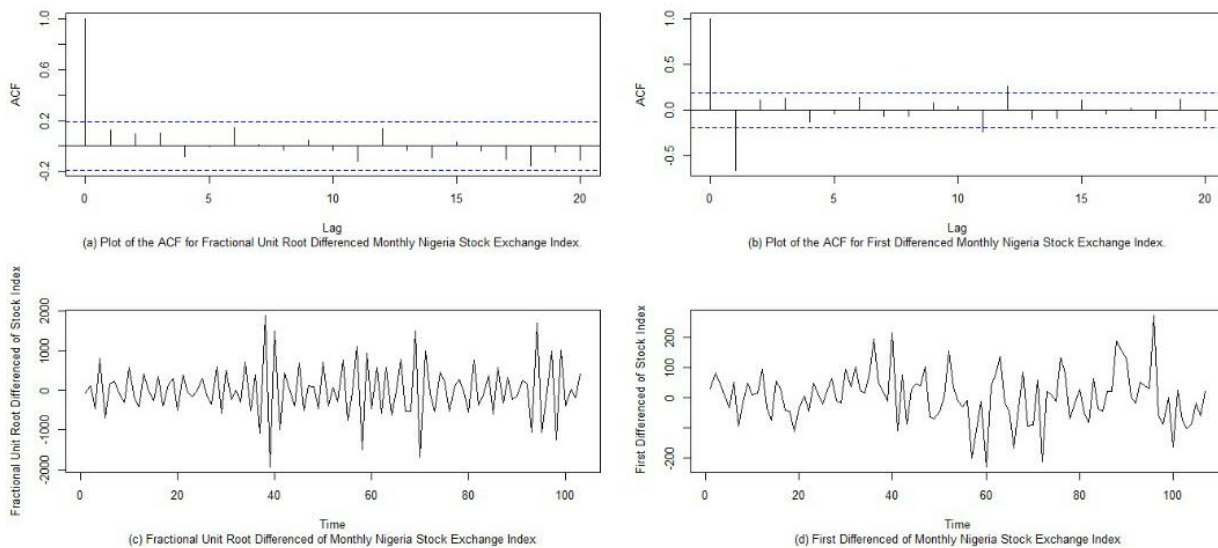


Figure 2: Time Series Plot and ACF for Both Fractional Unit Root Differenced and First Differenced Monthly Nigeria Stock Exchange Index (30 biggest stocks by market capitalization).

Table 1: Summary Statistics

Statistics/Test	Nigeria Stock Index
Mean	1374.20
Minimum	875.70
Maximum	2019.80
Standard Deviation	319.27
Skewness	0.2671
Kurtosis	-1.2309
Jarque-Bera	8.1020

Notes: The Jarque–Bera test corresponds to the test statistic for the null hypothesis of normality in the distribution of sample data.

Table 1 presents the summary statistics and normality test for the N30SPI index. The mean monthly value of the stock exchange for 30 biggest stocks by market capitalization is 1374.20 Naira. In finance, kurtosis is used as a measure of financial risk. It is possible for a stock market to have a low kurtosis which is good, but the overall standard deviation is high which is bad and vice-versa. The kurtosis of N30SPI index is negative -1.2309 and standard deviation or measure of volatility is 319.27. The two measure indicates that the Nigeria stock market with negative small kurtosis has small risk and is desirable for investors because there is small probability that the stock market would experience high volatility or risk. Similarly, the Jarque-Bera statistic shows that for the monthly stock exchange series, the null hypotheses of normality is rejected at the 0.05 significance level.

3.1 Unit Root and Long Memory Test

The KPSS unit root test, GPH long memory test and fractional unit root differencing value estimation for the N30SPI data are carried out and discuss in this section.

Table 2: Unit Root and Long memory Test

Test	N30SPI	FDN30SPI	FURDN30SPI
KPSS	0.7069(0.014)	2.4171(0.251)	5.1712(0.731)
GPH	4.6228(0.000)	4.3692(0.402)	8.4105(0.826)
d_{GPH}	1.245(0.984)	nil	nil

Note: p-values are in parenthesis (.), d_{GPH} is long memory based on GPH (1983), FDN30SPI and FURDN30SPI are First Differenced and Fractional Unit Root Differenced series.

The KPSS and GPH confirm the unit root and long memory at level series of the N30SPI as shown in Table 2. Both the null hypothesis of stationarity and no long memory are rejected because all the p-values are less than the 0.05 significance level. However, both test result indicates stationarity after FDN30SPI and FURD30SPI series are obtained. The estimator, d_{GPH} produce fractional unit root differencing value equal 1.245 which is in the interval of $1 < d < 2$. This estimator produces long memory value that can adequately eliminate unwanted noise signals in the stock index. Besides, the value confirms its strength in eliminating deterministic trend and can produce a series with less variability as shown in Figure 2(a) and (c).

3.2 The Identification and Estimation of ARIMA and ARFURIMA Models

This section discusses the identification, estimation and diagnostic tests of the mean model; ARIMA and ARFURIMA. The parameters of the ARIMA and ARFURIMA mean model will be estimated for the monthly N30SPI series in this section. The AIC, Log-likelihood value, φ , θ and diagnostic statistic estimates will be thoroughly examined due to their importance in deciding the appropriate model in this study.

3.2.1. The Identification and Estimation of ARIMA and ARFURIMA Models

The AIC of the candidate's model of ARIMA and ARFURIMA are displayed in Table 3 and are compared.

Table 3: AIC Values for Candidate Models

p,q	ARIMA	ARFURIMA
1,0	1257.03	1199.87
0,1	1257.28	1199.96
1,1	1258.14	1201.82
2,1	1260.04	1203.93
1,2	1259.96	1170.31

A comparison between all models shows that the ARIMA (1,1,0) and ARFURIMA (1,1.245,2) provides the minimum AIC and are in bold. This implies that the models are better fit to the stock index data. Next, the parameters for both models are estimated.

Table 4: Estimation of ARIMA(p,d,q) and ARFURIMA(p,d,q) with their Log-likelihood Values

Models	φ_1	θ_1	θ_2	Log-likelihood
ARIMA (1,1,0)	0.13(0.0955)	nil	nil	-626.52
ARFURIMA(1,1.245,2)	-0.87(0.0009)	-0.68(0.0008)	0.62(0.0008)	-580.16

Note: p-values are in parenthesis

The results of the estimated parameters of ARIMA and ARFURIMA model for the studied series and their log-likelihood values are given in Table 4. All the parameters in the ARFURIMA model are significant due to minimum standard error values. The ARFURIMA has the maximum log-likelihood values compared to the ARIMA model. This happens because the interminable long memory fractional filter in ARFURIMA have eliminated large inherent noise signals in the stock index series. Based on these observations, therefore, between the two filters in ARIMA and ARFURIMA, the best filter method is the interminable long memory fractional filter that uses fractional unit root differencing transformation.

3.2.2. The Diagnostic Tests of ARIMA and ARFURIMA Models

The diagnostic tests are the serial correlation analyses of the residuals of time series models. In this study, the Jarque-Bera and ARCH-LM test are used for testing normality and homoscedasticity of residuals of models.

Table 5: The ARIMA(p,d,q) and ARFURIMA(p,d,q) and Diagnostic Tests.

Models	Normality Test	ARCH-LM Test
ARIMA (1,1,0)	4.56(0.1023)	39.25(0.5041)
ARFURIMA(1,1.245,2)	0.93(0.6271)	35.00(0.6945)

Note: p-values are in parenthesis and the null hypothesis is rejected at 5% levels, normality of models residuals are tested by using Jarque-Bera statistics and ARCH-LM test at lag 40

The results of the serial correlation analyses for ARIMA and ARFURIMA are shown in Table 5. Results indicate evidence of normality and homoscedasticity in both the ARIMA and ARFURIMA model residuals due to non-zero p-values of Jarque-Bera and ARCH-LM test statistics. However, as we can see the ARFURIMA test statistic p-values: 0.6271 for normality test and 0.6945 for ARCH-LM test are greater than the p-values which are 0.1023 and 0.5041 for ARIMA model residuals. Therefore, we can state that the ARFURIMA model residuals are better and similar to a white noise process.

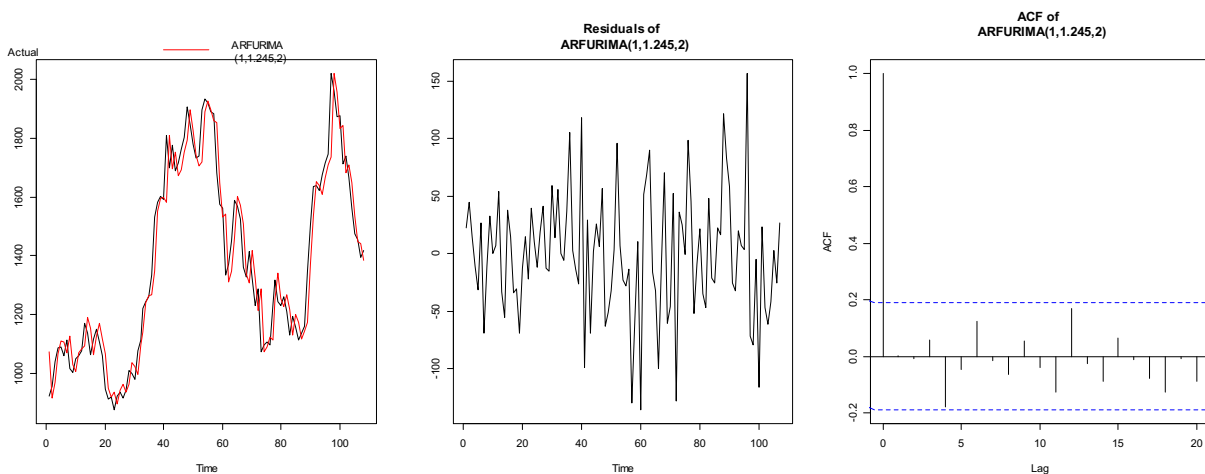


Figure 3: Diagnostic Plots of the ARFURIMA(1,1.245,2) Model Fitted to N30SPI.

Furthermore, the diagnostic plots of the in-sample forecasts are given in Figure 3. The plots show that in-sample forecasts coincide with the actual values observed for the ARFURIMA models. The ARFURIMA model

residuals are not trendy and ACF of the residuals are smaller and within the confidence limit indicating stationarity. This implies that ARFURIMA is good in terms of fit.

3.3 Diebold and Mariano Test for ARIMA and ARFURIMA Models

The performance of forecasts is carried out for ARIMA and ARFURIMA models. Gil-Alana and Toro (2002) had shown the power of mean models to forecast future values. They used ME, RMSE, MPE, Mean Square Error (MSE) and Mean Absolute Percentage Error (MAPE) as measures of accuracy to evaluate the forecast results.

In this study, ME and RMSE are used to evaluate the forecast performance. Results given in Table 6 show that the ARFURIMA model produces a better forecast with minimum ME and RMSE.

Similarly, the Diebold-Mariano test compares the forecast accuracy of two forecast methods. The results as shown in Table 7 is indicating that the ARFURIMA is better in forecasting monthly Nigeria stock index. The null hypotheses ARIMA is greater than ARFURIMA model in terms of the forecast is rejected at the 0.05 significance level.

Therefore, the plots and forecast of the 24 months (January 2019 to December 2020) out-sample for the series analyzed by using the ARFURIMA model are presented in Figure 4 and Table 8 respectively. The plots and forecast values show that out-sample forecasts decrease. This implies that the ARFURIMA is the appropriate model for modelling and forecasting the monthly N30SPI data used in this study. This is because the forecast decreasing trend indicate what is now observed in the Nigerian economy due to COVID-19 impact.

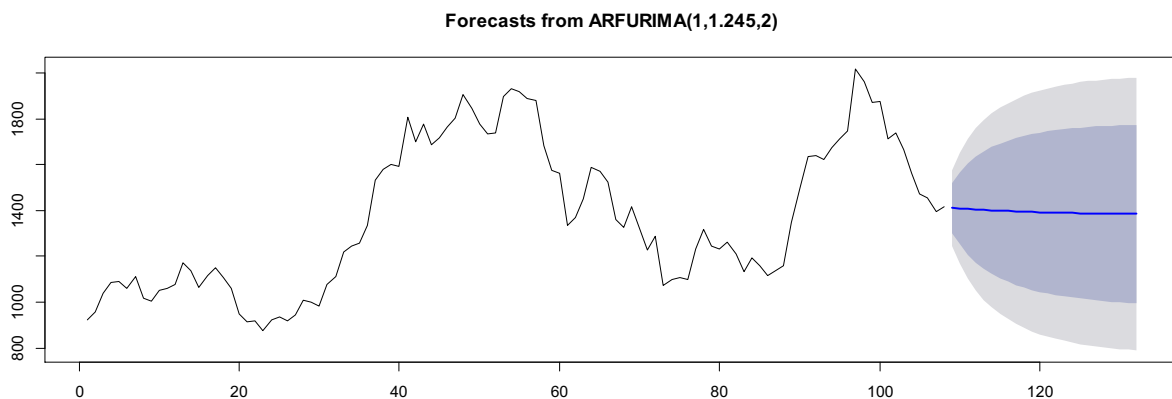


Figure 4: Forecasts of N30SPI by Using ARFURIMA(1,1.245,2).

Table 6: Forecasts Accuracy Values of ARIMA and ARFURIMA Model

Models	ME	RMSE
ARIMA (1,1,0)	4.0157	84.0676
ARFURIMA(1,1.245,2)	2.7740	83.5993

Table 7: Diebold and Mariano Test for ARIMA vs ARFURIMA Model

Variable	DM Statistic
N30SPI	-7.6905(0.0000)

DM=Diebold and Mariano test statistic and p-values are in parenthesis

Table 8: Forecast and 95% Confidence Intervals of the ARFURIMA (1,1.245,2) Models

Year	Months	Forecast Values	95% Confidence Lower	95% Confidence Higher
2019	January	1410.52	1244.35	1576.70
	February	1410.19	1168.44	1651.94
	March	1407.36	1102.07	1712.66
	April	1404.88	1051.13	1758.62
	May	1402.70	1010.33	1795.08
	June	1400.79	976.81	1824.77
	July	1399.09	948.84	1849.34
	August	1397.58	925.23	1869.92
	September	1396.21	905.14	1887.28
	October	1394.98	887.94	1902.02
	November	1393.86	873.12	1914.59
	December	1392.84	860.32	1925.36
2020	January	1391.90	849.21	1934.6
	February	1391.04	839.53	1942.54
	March	1390.25	831.09	1949.40
	April	1389.51	823.71	1955.31
	May	1388.83	817.24	1960.43
	June	1388.20	811.55	1964.86
	July	1387.61	806.54	1968.69
	August	1387.07	802.12	1972.01
	September	1386.55	798.21	1974.89
	October	1386.07	794.76	1977.39
	November	1385.63	791.69	1979.56
	December	1385.20	788.96	1981.44

4. Discussion

In this study, the modelling and forecasting of monthly Nigeria stock price index representing the thirty (30) biggest stocks by market capitalization was carried out. This type of stock is found to be a unit root process as shown in Olayiwola et al., (2016). However, the stock price is found to exhibit a type of long memory characteristics in the current work which is described interminable long memory. The GPH test results confirmed the interminable long memory characteristics observed and the long memory estimator, d_{GPH} , produce fractional unit root differencing value equal 1.245 which is in the interval of $1 < d < 2$. Applying the first differencing transformation based on $d = 1$, spikes in many lags were seen in the ACF of the differenced stock price indicating non-stationarity. This contradicts the position of Haldrop and Vera-Valdes (2017) who suggested that time series to be considered for



modelling and forecasting should be stationary. However, a stationary series was obtained when fractional unit root differencing with $d=1.245$ carried out. For comparison and robustness, the ARIMA and ARFURIMA model were used to study the monthly stock price index. After several ARIMA and ARFURIMA were identified and compared, ARFURIMA (1,1.245,2) provides the minimum information criteria, AIC. Similarly, the parameters of the estimated ARIMA and ARFURIMA model were compared based on parameters standard errors and log-likelihood values. The ARFURIMA indicate significant parameter values due to minimum standard error values. Also, the ARFURIMA has the maximum log-likelihood values compared to the ARIMA model. This achieved due to the interminable long memory fractional filter in ARFURIMA have removed large inherent noise signals in the stock index series. The superiority of the ARFURIMA is shown in the model adequacy test statistic's p-values. The 0.6271 for normality test and 0.6945 for ARCH-LM test for the ARFURIMA are greater than the p-values which are 0.1023 and 0.5041 for the ARIMA model residuals. Similar results also noticed in the ACF of the residuals of ARFURIMA model because the autocorrelation values are smaller and within the confidence, limit indicating white noise status of the model's residuals.

Finally, in the current work, the ARFURIMA model produces a better forecast with minimum ME and RMSE while ARIMA is shown to be better in Emenike (2008) using similar accuracy measures when compared to another mean model. Similar results were obtained in the Diebold and Mariano prediction accuracy test of the ARIMA and ARFURIMA method.

5. Conclusion

Differencing the Nigeria monthly stock price index that represents thirty (30) biggest stocks by market capitalization by using the fractional unit root differencing, the series is stationary. This set the stage for studying the target series by using the ARFURIMA methods. First, the results of the models' identification show that ARFURIMA is the best. Secondly, the results of the serial correlation analysis show that the ARFURIMA model residuals are better and similar to a white noise process. It can be stated that the ARFURIMA model achieved a special status every mean model strived to attain in modelling and forecasting time series. Secondly, the in-sample forecasts coincide with the actual values for the ARFURIMA models. This show that the ARFURIMA is good in terms of fit, and is the appropriate model for the stock index analyzed in this study. The out-sample forecast using the ARFURIMA model show decreasing stock prices will be obtained in the future. This implies that based on the range of data used, the ARFURIMA is the appropriate model for modelling and forecasting the monthly N30SPI data. This is because the forecast decreasing trend captured by the ARFURIMA model indicate the unexpected calamity now witnessed in the Nigerian economy and other economies due to Corona Virus 2019 (COVID-19) pandemic impact. Finally, this study was designed and conducted to bring to the notice of time series analyst the importance of chosen appropriate transformation methods for eliminating unwanted time-series components such as trend, long memory and interminable long memory. Specifically, when a time series is known to have fractional unit root differencing values in the interval of $1 < d < 2$ applying first differencing as used by many researchers may results in under-differencing, estimation of weak models and unreliable forecasts.

Acknowledgement

The authors are grateful to Laboratory for Time Series Modeling, Analysis, R Software and Financial Research of Nigeria (LATSMARF-RESEARCH-IN-NIGERIA) for developing R package used for analysis, providing technical assistance and using their laboratory facilities.

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