



Economic Order Quantity Model for Delayed Deteriorating Items with Time-Dependent Linear Declining Demand And its Sensitivity Analysis

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Abstract

The present study investigate an Economic Order Quantity (EOQ) Inventory model for delayed (non-instantaneous) deteriorating items is built on the assumption that the demand rate before deterioration begins is a linear declining function of time, while the demand rate after deterioration sets in becomes constant until the inventory is completely depleted. The optimal cycle length yields the lowest total inventory cost. By minimizing the total inventory cost, the model is solved analytically which results in the best cycle length being determined. Moreover, to demonstrate how changes in some model value parameters affect the model, we perform a sensitivity analysis.

Keywords: EOQ, Inventory, Delayed deterioration, linear declining demand

1.0 Introduction

Inventory can be defined as the stock of goods, commodities, and other economic resources that are stored or preserved to ensure the smooth and efficient running of businesses. Inventory may include stationeries, petroleum products, raw materials, furniture, machinery, blood in blood banks, and so on. The purpose of holding inventories in a manufacturing firm is to facilitate a constant and smooth rate of production for the firm which may include holding stocks of raw materials and semi-finished goods so that the production is not interrupted even if the industrial units supplying the materials are frequently troubled by power shortages, transport bottlenecks, labor unrest and so on. On the other hand, a company holds stocks of finished goods so that it can meet excess demand, out of the inventories accumulated during a period of low demand. A retail outlet holds stock to meet its usual day to day demand while inventories such as furniture, electronics, stationeries, drugs are held to meet the social, physical, educational, and other needs of the society.

The study of the inventory model comes into force in 1915. Harris [1] was the first mathematician to study an inventory problems. He developed the simple but famous EOQ formula that was also derived, apparently independently, by Wilson [2]. Gradually, the demand for goods may vary with time or with price or with the instantaneous inventory level displayed in a market. In recent years, inventory modelers are working for finding the economic replenishment policy for an inventory system having a time-dependent demand pattern. Silver and Meal [3] first developed a heuristic approach to determine EOQ in the general case of a time varying-demand pattern. Donaldson [4] first constructed a model on coming out with a fully analytic solution of the inventory replenishment problem with a linear trend in demand pattern over a finite-time horizon. Musa and Sani [5] constructed an inventory model of delayed deteriorating items under permissible delay in payment. Musa and Sani [6] developed an EOQ model for delayed deteriorating items. with linear time-dependent holding costs. Khanra et al. [7] developed an inventory model considering time-quadratic demand rate. During a delay period (or trade credit



period) suppliers usually offer their retailers a certain credit period with interest during the permissible delay period. Goyal [8] first developed the EOQ model under the conditions of permissible delay in payment. Yahaya, and Abubakar. [9] Developed an Economic Order Quantity for Delayed Deteriorating items with Time-Dependent Exponential Declining Demand. Adamu and Musa. [9] Determination of the Economic Order Quantity for Delayed Deteriorating Items with Linear Trended Demand and Partial Backlogging. Adamu and Musa. [10] Developed an EOQ Model for Delayed Deteriorating Items with Quadratic Time-Dependent Holding Cost and Backordering. Bashir and Musa, in [11] Production-Inventory Model of Delayed Deteriorating Items with Quadratic and Price Dependent Demands and Backordering. In this paper, an inventory model for delayed deteriorating items with linear time-dependent holding costs and back-ordering is developed. The retailer in this situation does not allow for back-ordering. The items back ordered are settled first when a new replenishment account is received.

1.1 Assumption and Notations

1.1.1 Assumptions

The demand rate is deterministic and is an linear declining function of time before deterioration sets in, i.e. in the interval $0 \leq t \leq T_1$

The demand rate is constant when deterioration sets in the interval, $T_1 \leq t \leq T$

The deteriorating rate is considered as constant, $\theta(t) = \theta$, $0 < \theta < 1$

Shortages are not allowed

Replenishment is instantaneous

Lead time is zero

1.1.2 Notations

$P_1(t)$ The Linear demand rate (units per unit time) before deterioration set in,

$$P_1(t) = \alpha_1 + \alpha_2 t$$

P_2 The demand rate (units per unit time) when the deterioration sets in

T_1 The time when the deterioration begins

T_2 The period of deterioration between T and T_1

T The inventory cycle length that is the time interval between two

C The unit cost of the item (Naira per unit)

i Inventory carrying charge per unit time (Naira per unit time)

A The fixed ordering or replenishment cost (Naira per order)

I_0 The initial inventory

$I(t)$ The inventory level at any time t before deterioration begins

I_a The inventory level at the time deterioration begins

$I_a(t)$ The inventory level at any time t after deterioration sets in

T_d Total demand throughout deterioration $[T_1, T]$

$TC(T)$ Total inventory cost per unit time (Naira per unit time)

Mathematical Formulation and Solution

Diagrammatic representation of the model

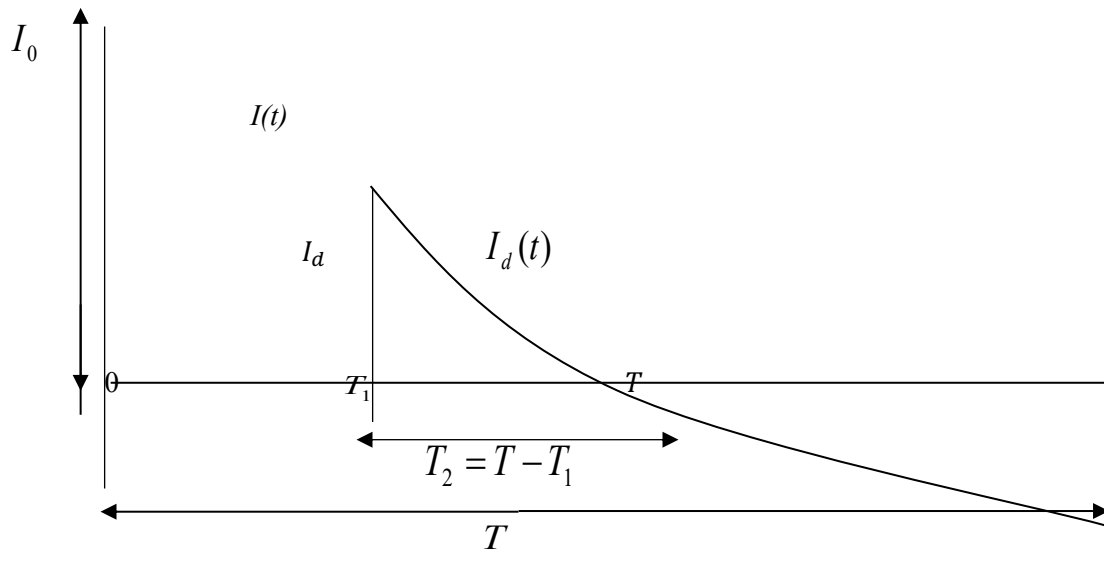


Figure 1: Inventory depletion in a delayed deterioration situation

The inventory status before the deterioration sets in is governed by the differential equation:

$$\frac{dI(t)}{dt} = -P_1(t), \quad 0 \leq t \leq T_1. \quad (1)$$

The differential equation representing the status of the inventory when the deterioration sets in is given by:

$$\frac{dI(t)}{dt} + \theta I_d(t) = -P_2(t), \quad T_1 \leq t \leq T. \quad (2)$$

Equation (1) is first-order variable separable, thus,

$$\frac{dI(t)}{dt} = -(\alpha_1 + \alpha_2 t), \quad \text{where } P_1(t) = (\alpha_1 + \alpha_2 t),$$

$$I(t) = - \int (\alpha_1 + \alpha_2 t) dt,$$

$$I(t) = -\alpha_1 t - \frac{\alpha_2 t^2}{2} + C. \quad (3)$$

Substituting the boundary condition, at $t = 0$, $I(t) = I_0$ in equation (3), we get $I_0 = C$ so that

$$I(t) = -\alpha_1 t - \frac{\alpha_2 t^2}{2} + I_0,$$

$$I(t) = I_0 - \alpha_1 t - \frac{\alpha_2 t^2}{2}. \quad (4)$$

Putting the condition $t = T_1$, $I(t) = I_d$ in equation (4) we get,

$$I_d = I_0 - \alpha_1 T_1 - \frac{\alpha_2 T_1^2}{2},$$

$$I_0 = I_d + T_1 \left(\alpha_1 + \frac{\alpha_2 T_1}{2} \right). \quad (5)$$

Substituting (5) in (4) we get,

$$I(t) = I_d + T_1 \left(\alpha_1 + \frac{\alpha_2 T_1}{2} \right) - \alpha_1 t - \frac{\alpha_2 t^2}{2},$$

$$I(t) = I_d + T_1 \alpha_1 - \alpha_1 t + \frac{\alpha_2 T_1^2}{2} - \frac{\alpha_2 t^2}{2},$$

$$I(t) = I_d + \alpha_1 (T_1 - t) + \frac{\alpha_2}{2} (T_1^2 - t^2). \quad (6)$$

Equation (2) is a first-order linear differential equation with integrating factor $I.F = e^{\int \theta dt} = e^{\theta t}$. The solution of (2) is thus given by:

$$I.F I_d(t) = \int I.F Q(t) dt, \text{ where } K(t) = \theta \text{ and } Q(t) = -P_2,$$

$$I_d(t) e^{\theta t} = -P_2 \int e^{\theta t} dt,$$

$$= I_d(t) e^{\theta t} = \frac{-P_2 e^{\theta t}}{\theta} + C_1.$$

Where C_1 is arbitrary constant

$$I_d(t) = \frac{-P_2}{\theta} + C_1 e^{-\theta t}. \quad (7)$$

Putting the condition, at $t = T_1$, $I_d(t) = I_d$ in equation (7) we get,

$$I_d = \frac{-P_2}{\theta} + C_1 e^{-\theta T_1},$$

$$= C_1 e^{-\theta T_1} = I_d + \frac{P_2}{\theta},$$

$$C_1 = \left(I_d + \frac{P_2}{\theta} \right) e^{\theta T_1}. \quad (8)$$

Putting equation (8) in (7) we get,

$$\begin{aligned}
 I_d(t) &= \frac{-P_2}{\theta} + \left(I_d + \frac{P_2}{\theta}\right) e^{\theta T_1} \cdot e^{-\theta t}, \\
 &= \frac{-P_2}{\theta} + \left(I_d + \frac{P_2}{\theta}\right) e^{\theta(T_1-t)}, \\
 &= \frac{-P_2}{\theta} + I_d e^{\theta(T_1-t)} + \frac{P_2}{\theta} e^{\theta(T_1-t)}, \\
 I_d(t) &= I_d e^{\theta(T_1-t)} + \frac{P_2}{\theta} (e^{\theta(T_1-t)} - 1). \tag{9}
 \end{aligned}$$

Putting the condition at $t = T$, $I_d(t) = 0$ in equation (9) we get,

$$\begin{aligned}
 0 &= I_d e^{\theta(T_1-T)} + \frac{P_2}{\theta} (e^{\theta(T_1-T)} - 1) \\
 I_d e^{\theta(T_1-T)} &= -\frac{P_2}{\theta} (e^{\theta(T_1-T)} - 1) \\
 I_d &= -\frac{P_2}{\theta} (e^{\theta(T_1-T)} - 1) e^{-\theta(T_1-T)} \\
 &= -\frac{P_2}{\theta} + \frac{P_2}{\theta} e^{-\theta(T_1-T)} \\
 I_d &= \frac{P_2}{\theta} (e^{-\theta(T_1-T)} - 1) \\
 I_d &= -\frac{P_2}{\theta} (1 - e^{\theta(T-T_1)}). \tag{10}
 \end{aligned}$$

Substituting equation (10) into equation (6) we get,

$$I(t) = -\frac{P_2}{\theta} (1 - e^{\theta(T-T_1)}) + \alpha_1(T_1 - t) + \frac{\alpha_2}{2} (T_1^2 - t^2). \tag{11}$$

Again substituting (10) into equation (9) we get,

$$\begin{aligned}
 I_d(t) &= -\frac{P_2}{\theta} (1 - e^{\theta(T-T_1)}) e^{\theta(T_1-t)} + \frac{P_2}{\theta} (e^{\theta(T_1-t)} - 1) \\
 I_d(t) &= -\frac{P_2}{\theta} e^{\theta(T_1-t)} + \frac{P_2}{\theta} e^{\theta(T-T_1)} \cdot e^{\theta(T_1-t)} + \frac{P_2}{\theta} e^{\theta(T_1-t)} - \frac{P_2}{\theta} \\
 I_d(t) &= -\frac{P_2}{\theta} e^{\theta(T_1-t)} + \frac{P_2}{\theta} e^{\theta(T-t)} + \frac{P_2}{\theta} e^{\theta(T_1-t)} - \frac{P_2}{\theta} \\
 I_d(t) &= \frac{P_2}{\theta} (e^{\theta(T-t)} - 1). \tag{12}
 \end{aligned}$$

The total demand throughout deterioration $[T_1, T]$ is $T_d = P_2 \times (T - T_1) = P_2 T_2$

The number of items in the inventory that deteriorated

$$N_d = I_d - P_2 T_2. \quad (13)$$

Substituting for I_d from equation (10) into equation (13) we get,

$$\begin{aligned} N_d &= -\frac{P_2}{\theta}(1 - e^{\theta(T-T_1)}) - P_2 T_2 \\ &= -\frac{P_2}{\theta} + \frac{P_2}{\theta} e^{\theta(T-T_1)} - P_2 T_2 \\ &= -\frac{P_2}{\theta} [1 - e^{\theta(T-T_1)} + \theta T_2] \\ N_d &= -\frac{P_2}{\theta} [1 - e^{\theta(T-T_1)} + (T - T_1)\theta]. \end{aligned} \quad (14)$$

Computation of variable cost

The total inventory (variable) cost is made up of the sum of the following inventory-related costs:

- i. Ordering Cost, $O_c = A$
- ii. Inventory holding (carrying) cost, H_c
- iii. Cost of deteriorated items CN_d

These costs are calculated separately and then sum up to get the total variable cost.

The inventory holding (carrying) cost, H_c

$$\begin{aligned} H_c &= ic \int_0^{T_1} I(t) dt + ic \int_{T_1}^T I_d(t) dt \\ &= ic \int_0^{T_1} -\frac{P_2}{\theta}(1 - e^{\theta(T-T_1)}) + \alpha_1(T_1 - t) + \frac{\alpha_2}{2}(T_1^2 - t^2) dt + ic \int_{T_1}^T \frac{P_2}{\theta}(e^{\theta(T-t)} - 1) dt \\ &= ic \int_0^{T_1} \left(-\frac{P_2}{\theta} + \frac{P_2 e^{\theta(T-T_1)}}{\theta} + \alpha_1 T_1 - \alpha_1 t + \frac{\alpha_2 T_1^2}{2} - \frac{\alpha_2 t^2}{2} \right) dt + ic \int_{T_1}^T \left(\frac{P_2 e^{\theta(T-t)}}{\theta} - \frac{P_2}{\theta} \right) dt \\ &= \frac{-icP_2}{\theta} \int_0^{T_1} dt + \frac{icP_2 e^{\theta(T-T_1)}}{\theta} \int_0^{T_1} dt + ic\alpha_1 T_1 \int_0^{T_1} dt - ic\alpha_1 \int_0^{T_1} t dt + \frac{ic\alpha_2 T_1^2}{2} \int_0^{T_1} dt - \frac{ic\alpha_2}{2} \int_0^{T_1} t^2 dt \end{aligned}$$

$$\begin{aligned}
 & + \frac{icP_2}{\theta} \int_{T_1}^T e^{\theta(T-t)} dt - \frac{icP_2}{\theta} \int_{T_1}^T dt \\
 & \quad - \frac{icP_2 T_1}{\theta} + \frac{icP_2 T_1 e^{\theta(T-T_1)}}{\theta} + ic\alpha_1 T_1^2 - \frac{ic\alpha_1 T_1^2}{2} + \frac{ic\alpha_1 T_1^3}{2} - \frac{ic\alpha_2 T_1^3}{6} - \frac{icP_2}{\theta^2} + \frac{icP_2 e^{\theta(T-T_1)}}{\theta^2} \\
 & - \frac{icP_2 T}{\theta} + \frac{icP_2 T_1}{\theta} \\
 & = \frac{icP_2 T_1}{\theta} \left\{ e^{\theta(T-T_1)} + \frac{\theta\alpha_1 T_1}{P_2} - \frac{\theta\alpha_1 T_1}{2P_2} + \frac{\theta\alpha_1 T_1^2}{2P_2} - \frac{\alpha_2 T_1^2}{6P_2} - \frac{1}{\theta T_1} + \frac{e^{\theta(T-T_1)}}{\theta T_1} - \frac{T}{T_1} \right\}. \quad (15)
 \end{aligned}$$

Cost of deteriorated items

$$\text{Cost of deteriorated items, } CN_d = C \left[-\frac{P_2}{\theta} [1 - e^{\theta(T-T_1)} + (T - T_1)\theta] \right]. \quad (16)$$

Where C is a unit cost of items.

The total variable cost per unit time is given by

$$TC(T) = \frac{1}{T} \{ \text{Inventory ordering cost, } O_c + \text{inventory holding, } H_c + \text{cost of deteriorated items, } CN_d \}$$

$$\frac{1}{T} \{ O_c + H_c + CN_d \}$$

$$\frac{A}{T} + \frac{icP_2 T_1}{T\theta} \left\{ e^{\theta(T-T_1)} + \frac{\theta\alpha_1 T_1}{P_2} - \frac{\theta\alpha_1 T_1}{2P_2} + \frac{\theta\alpha_1 T_1^2}{2P_2} - \frac{\alpha_2 T_1^2}{6P_2} + \frac{1}{\theta T_1} - \frac{e^{\theta(T-T_1)}}{\theta T_1} - \frac{T}{T_1} \right\} + \frac{C}{T} \left[-\frac{P_2}{\theta} [1 - e^{\theta(T-T_1)} + (T - T_1)\theta] \right]. \quad (17)$$

We aim to optimize the variable cost per unit time. The necessary and sufficient conditions that minimize our $TC(T)$ for a given value of T are respectively: $\frac{dT C(T)}{dT} = 0$ and $\frac{d^2 TC(T)}{dT^2} > 0$.

$$\begin{aligned}
 \frac{dT(T)}{dT} &= -\frac{A}{T^2} + \frac{icP_2 T_1}{\theta T^2} \left\{ (T\theta - 1)e^{\theta(T-T_1)} - \frac{\theta\alpha_1 T_1}{2P_2} + \frac{T_1^2}{2P_2} \left(\frac{\alpha_2}{3} - \theta\alpha_1 \right) - \frac{1}{\theta T_1} - \frac{1}{\theta T_1} (T\theta - 1)e^{\theta(T-T_1)} \right\} + \\
 \frac{C}{T^2} &\left[-\frac{P_2}{\theta} (-1 + (T\theta - 1)e^{\theta(T-T_1)} + T\theta - (T - T_1)\theta) \right] = 0. \quad (18)
 \end{aligned}$$

Multiply equation (18) by T^2 and equating to zero

$$-A + \frac{icP_2 T_1}{\theta} \left\{ (T\theta - 1)e^{\theta(T-T_1)} - \frac{\theta\alpha_1 T_1}{2P_2} + \frac{T_1^2}{2P_2} \left(\frac{\alpha_2}{3} - \theta\alpha_1 \right) - \frac{1}{\theta T_1} - \frac{1}{\theta T_1} (T\theta - 1)e^{\theta(T-T_1)} \right\} + C \left[-\frac{P_2}{\theta} (-1 + (T\theta - 1)e^{\theta(T-T_1)} + T\theta - (T - T_1)\theta) \right] = 0. \quad (19)$$

We use (19) to determine the best cycle length T which minimizes the total inventory cost. The economic order quantity corresponding to the optimal cycle length $T = T^*$ can be computed as follows: $EOQ = \text{Total demand in the period } [0, T_1] \text{ before deteriorated sets in} + \text{Total demand over the deterioration period } [T_1, T] + \text{Number of items that deteriorated}$

$$P_1(t)T_1 + P_2T_2 + N_d$$

$$(\alpha_1 + \alpha_2t)T_1 + P_2(T - T_1) - \frac{P_2}{\theta} [1 - e^{\theta(T-T_1)} + (T - T_1)\theta]$$

$$EOQ = (\alpha_1 + \alpha_2t)T_1 + \frac{P_2}{\theta} [e^{\theta(T-T_1)} - 1]. \tag{20}$$

3.0 Numerical Examples 1

The solutions of five different numerical examples representing the application of the model are given in the table below :

3.1 Numerical example

S/N	A (Naira)	C	P_1 (Units)	P_2 (Units)	i	T_1	T_2	θ	α_1	α_2	T	$TC(T)$	EOQ (Units)
1	65	30	50	40	0.018	0.0420	0.0431	0.2	600	550	0.6019	901.800	162
2	55	20	40	30	0.013	0.0162	0.0435	0.1	250	200	0.5301	482.379	49
3	75	40	65	50	0.016	0.0672	0.0622	0.3	750	650	0.5573	2250.34	283
4	85	55	70	60	0.014	0.0820	0.0532	0.4	500	600	0.5285	4283.08	312
5	95	40	85	70	0.015	0.0840	0.0542	0.5	550	650	0.5489	6154.61	355

For the numerical illustration of the developed model, the values of various parameters in proper units can be taken as follows:

$A = 65, C = 30, P_2 = 40$. Solving Equation (19) with the above parameters, we obtain $T^* = 0.6019$. On substitution of the optimal value T^* in Equations (17) and (20), we obtain the minimum total cost per unit time $TC^* = 901.800$ and the economic order quantity $EOQ = 162$ respectively. We now study the effect of changes in the values of the system parameters and A, P_1, C on the optimal cost and number of reordering. The sensitivity analysis is performed by changing each of the parameters by 50%, 25%, -25% -50%, and taking one parameter at a time and keeping the remaining parameters un-changed.

The analysis is based on Example 1 and the results are shown in **Table 2**.

3.2 Sensitivity analysis

In this part of the paper we study the effect of changes in the values of the parameters A , C , and P_2 on the optimal cycle length (T) and optimal variable cost (TC) and Economic Order Quantity (EOQ). The sensitivity analysis is performed by changing some of the parameters at a time and keeping the remaining parameters unchanged. Using Maple Software (2017).

4.0 Table 2: Results of Sensitivity Analysis

Parameter	% change in parameter	T	% change in parameter	$TC(T)$	% change in parameter	EOQ (unit)	% change in parameter
A	+50 (120)	0.1815	-69.7956	6322.40	601.0867	162	-----
	+25 (90)	0.4108	-31.7494	2063.27	128.7946	184	13.5802
	-25 (40)	0.7930	31.7494	438.43	-51.3827	208	28.3950
	-50 (05)	0.9842	63.5155	99.150	-89.0053	231	42.5925
P_2	+50 (90)	0.7518	24.9044	928.93	3.008427	217	33.9506
	+25 (65)	0.7057	17.2453	879.19	-2.50720	213	31.4814
	-25 (15)	0.1524	-74.6801	3048.98	238.0993	180	11.1111
	-50 (-10)	1.9505	224.0571	455.73	-49.4644	153	-5.5555
C	+50 (80)	0.9125	51.6032	279.79	-68.9742	162	-----
	+25 (55)	0.8275	37.4813	482.98	-46.4426	185	14.1975
	-25 (05)	-1.8826	-412.77	-364.50	-105.1499	208	28.3950
	-50 (-20)	1.8442	206.39	449.99	-50.1009	231	42.5925

4.1 Discussion on the results of sensitivity analysis

The results obtained from the sensitivity analysis as presented in Table 2 are discussed below:

(i) Increase in the parameter A . by (50%) decreases the optimal cycle length (T) and also increases in the optimal total variable cost (TC) but the EOQ remained unchanged. Similarly, a decrease in parameter A . by (25%) increases the optimal cycle length (T) and also decreases as the optimal total variable cost, (TC) and increases the EOQ. This also shows that the model is highly sensitive to change in parameter A . (ii) Increase in the parameter P_2 , by (50%) increases the optimal cycle length (T) as well as the optimal total variable cost (TC) and very slightly increases the EOQ. Otherwise decrease in the parameter P_2 , by (25%) also decrease the optimal cycle length (T) and increase the optimal total variable cost (TC) as well as the EOQ. This shows that the model is moderately sensitive to changes in the parameter P_2 . (iii) An increase in parameter C , by (50%) increases the optimal cycle length (T) and also decreases the optimal total variable cost (TC) but the EOQ remained on change. Otherwise, an increase in the parameter C . by (25%) increases the optimal cycle length (T) and also decrease the optimal total variable cost (TC) and also increase the EOQ, which is very low sensitivity. Similarly decrease in parameter C , by (50%) increases the optimal cycle length (T) and also decreases the optimal total variable cost (TC) and increases the EOQ. This shows that the model is highly sensitive to changes in parameter C .



5.0 Conclusion

In this study, we developed an inventory model that applies to a business environment that considers the fact that deterioration of the storage item is delayed (non-instantaneous) and in which the demand rate before deterioration begins is a linear declining function of time and when deterioration sets in, it is constant up to the end of the cycle. The model does not allow for shortages. The model was solved analytically to determine the minimum total inventory cost, (TC) the optimal cycle length (T), and the Economic Order Quantity (EOQ). Five different parameter values are taken to provide numerical examples on the application of the model. The result obtained from the numerical examples indicates the validity and stability of the model. Finally sensitivity analysis was carried out to show how change in some values parameters affect the model.

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