

# Inventory Ordering Policy of Delayed Deteriorating Pharmaceutical Items with Two-Phase Demand Rates Under Preservation Technology Investment in Healthcare Environment

Yahaya Ahmed<sup>1,\*</sup> and Abubakar Musa<sup>2</sup>

<sup>1</sup>Department of Mathematics, Kano State College of Education and Preliminary Studies,  
Kano State, Nigeria

<sup>2</sup>Department of Mathematics, Aliko Dangote University of Science and Technology, Wudil,  
Kano State, Nigeria

\*Corresponding author email: yahayacas23@gmail.com; Phone number: 08076204866

## Abstract

*This study proposes an inventory ordering policy for delayed deteriorating pharmaceutical products with two-phase demand rates in healthcare environment. In most cases, the market demand rate for newly introduced pharmaceutical products such as drugs and other healthcare consumables increases with time. The demand rate of such items remains constant for some period of time, and shortly after, when the products become popular in the pharmacy or market, on the identification of the quality of the product and through advertisement, the demand rises. The model assumes constant demand in the first phase of the inventory system and linear time-dependent demand in the second phase. The rate of deterioration (which affect the efficacy of the items) can be reduced by means of effective preservation technology investment. The objective of the model is to find the optimal replenishment policy that minimizes the retailer's total cost at a reduced deteriorating rate due to investment in preservation technology. An iterative procedure is proposed to solve the model mathematical formulation and its application is validated with some numerical examples. A Sensitivity analysis is carried out to illustrate the effect of changes in the system parameters. The outcome of the sensitivity analysis reveals that investment in preservation technology has significant impact on the inventory policy. As more investment is made into the system, more items are purchased due to their freshness, as such, order quantity has to be increased to avoid shortages. Some other managerial decisions are also provided.*

**Keywords:** *Delayed Deterioration, Healthcare Environment, Inventory Ordering Policy, Preservation Technology, Two-phase demand.*

## 1. Introduction

Deterioration is the major factor that causes inventory loses. It is the main causative agent for increasing total inventory cost which in turns decreases the total profit. Deterioration refers to as decay, expiration, damage, spoilage, evaporation, obsolescence, pilferage, loss of utility or loss of marginal value of item that results in decreased of its usefulness or render it invalid (Rastogi and Singh, 2019). Nearly all products deteriorate over time. Pharmaceutical items such as tablets, caplets, ointments, injectables, ophthalmics, liquids, etc. have a specific period of lifetime and they do not deteriorate instantly, the phenomenon called “delayed deterioration”. Shah et al. (2021) reported that deterioration is influenced by factors such as: oxidation, microorganisms and enzymes. The action of microorganisms is influenced by some factors like, humidity, atmosphere and temperature. In order to curtail the negative effect of temperature on deteriorating items effective preservation technology like cooling system is needed to halt the deterioration process. Item's deterioration rate can be reduced by incorporating modern storage

technology such as, enhancing cooling system, modern packaging style, etc. Cooling keeps low temperature in stocked items consequently prevents chemical deterioration and microbial spoilage. Preservation technology can extend products life and provides benefits which result in increasing the total profit output at minimum total inventory cost. In pharmaceutical inventory system, spoilage or deterioration of pharmaceutical products and drugs refers to the changes in the physical and chemical properties in such a way that the formulation or therapeutic agent gets deteriorate and become unsuitable for use. Pharmaceutical product is a combination of two things: **Active drug** and **Formulation additives** (Mehendiratta, 2023). If there is any change in their efficacy, action, etc. it is said to be spoiled/deteriorated product which cannot be used. Mehendiratta (2023) outlined three factors that cause pharmaceutical spoilage/deterioration: **i) Physical Factors:** This refers to deterioration due to physical factors like, humidity, heat, temperature, evaporation, etc. **ii) Chemical Factors:** This is the deterioration as a result of chemical reactions like oxidation, reduction, hydrolysis, photolysis, ionization and so on. **iii) Microbial Factors:** This refers to deterioration due to the action of any microbial cell like Bacteria, fungi, moulds etc.

Some pharmaceuticals like Ophthalmic (ear/eye drops: Chloramphenicol, Tobramycin, Vigamox (moxifloxacin) e.tc) can be kept at room temperature for 28 days after opening. Reconstituted antibiotics (Amoxicillin, Erythromycin, Augmentin) once reconstituted need to be kept in refrigerator and be discarded after 1 to 2 weeks. Tablets (Leukeran, Alkeran, Ritonavir, VePesid), Injections (Insulin, Glucagon, Victoza, etc.) may be safe to store at room temperature for about a month, after that timeframe, refrigeration is essential. However, other pharmaceutical like anthrax immune globulin (Anthrasi), Carmustine wafer (Gliadel), Cholera(live)vaccine (Vaxchora), Dinoprostone vaginal insert (Cervidil), Dinoprostone vaginal suppository (Prostin), Varicella vaccine (Varivax) and so on, require intensive frozen storage (Source: drugs.com/medical-answers). Therefore, without preservation technology investment the inventory system containing this type of items could not be operated because it would be unprofitable.

Our overview to existing inventory literatures showed although a considerable number of research models have already been proposed in relation to deteriorating items, however, none has considered non-instantaneous deteriorating pharmaceutical inventory, with two phase demand pattern and preservation technology investment in healthcare environment. Thus, our model intends to answer the following questions:

- i).** How could two-phased demand influence ordering policy in pharmaceutical inventory system in healthcare environment?
- ii).** How preservation technology investment influences the decisions and provide benefits to pharmaceutical inventory system?

This model is applicable for pharmaceutical sector, in particular, healthcare pharmaceutical system to help suppliers and retailers make inventory decisions in their inventory system under the effect of optimal values. The policy combinations for healthcare pharmaceutical systems are generally designed to reduce the deterioration rate, the expenditures, and the total cost. The proposed model with cost functions yields the best operating plan to achieve the said objectives. The remainder of this paper is organized as follows: Section 2 provides general review on related literatures. Section 3 provides research gap and our contribution. Section 4 presents model description and formulation. Section 5 provides the optimality analysis of the model's solution. Section 6 provides solution algorithm to the model's optimal solution. Section 7 gives numerical examples to validate the model's solution. Section 8 provides the sensitivity analysis to study the effect of changes in the values of some sensitive system parameters. Section 9 provides general comments on the observation made from the results for the sensitivity analysis. Finally, Section 10 concludes the paper.

## 2. Related Works

The history of study of deteriorating inventory system can be traced back to 1963 when Ghare and Schrader first introduced the concept in their model of exponentially decaying inventory. After them, many researchers such as: Shah (1977), Dave and Patel (1981), Chung and Tin (1993), Wee (1995), Chang and Dye (1999) and so on came up with several modifications of the model under different conditions of deterioration rates, demands and holding costs. In these literatures, deteriorations were regarded as instantaneous but in reality it has been observed that deterioration of the product may start at certain period after production and it can be reduced due to retailer's investment in preservation technology.

Hsu et al. (2010) formulated the first preservation technology model for deteriorating inventory. The main objective of their paper was to obtain the retailer's replenishment and preservation technology investment policy that maximizes the retailer's unit time profit. The model was supported by graphical analysis approach to show the concavity of the objective function. However, they left the proof of existence and uniqueness of the optimal solution for future research. Huang et al (2011) complemented the shortcomings of Hsu et al. (2010) by proving that the optimal replenishment schedule not only exists but is unique for any given invested capital. In addition they showed that the retailer's unit time profit is a concave function of invested capital when the replenishment schedule is given. Dye (2013) studied the effect of preservation technology investment for inventory system of non-instantaneous deteriorating items. Kumar et al. (2014) studied an EOQ model for time-dependent demand and parabolic holding cost with preservation technology under partial backlogging for deteriorating items. The proposed model was developed to minimize the total inventory cost. Zhang et al. (2014) proposed an optimal pricing policy for deteriorating items with preservation technology investment. Mahata and Debnath (2022) extended Zhang et al (2014) by considering a single item profit maximization inventory problem with price dependent demand considering deterioration during carrying and preservation technology investment. The model considered two-level supply chain system taking into account deterioration during carrying of deteriorating items from supplier's warehouse to retailer's warehouse as well as the reduced deterioration in the retailer's warehouse due to preservation technology investment. The model maximizes the retailer's profit per unit time. A price dependent demand and storage time dependent holding cost was considered in the developed model. Priyamvada and Jaggi (2022) further extended Mahata and Debnath (2022) model by considering a price-dependent investment in preservation technology. They regarded the two assumptions made by Mahata and Debnath (2022) that: two different products having the same deteriorating rate but with different selling price have equal investment, and the investment in preservation technology to be taken per unit time irrespective the number of units held, as illogical assumptions. They stressed that not only the investment is per unit, but also depends on the selling price of that unit. De and Mallick (2020) studied the effect of price concession strategy and preservation technology on inventory system for decaying products with partial backlogging and price-stocked-dependent on demand. A retailer's inventory decisions with promotional efforts and preservation technology investments when the supplier offers quantity discounts was proposed by (shah et al. 2021).

The above literatures centered their finding on prize sensitivity of demand of the items. Several factors can influence the product demand. Mishra (2013) proposed deteriorating inventory model using preservation technology with salvage value and shortages. Singh and Rathore (2015) formulated an optimal payment policy with preservation technology investment with shortages under Trade Credit. The model considered linear time dependent demand rate where total cost and payment time were optimized. Mishra and Shaik (2015) proposed optimal policies for deteriorating items with preservation and maintenance management when demand is trade credit sensitive. Hishamuddin et al. (2020) came up with improved preservation technology for non-instantaneous deteriorating inventory using boundary condition estimation and inventory level dependent demand.

Product quality is centroid of any inventory system. In pharmaceutical system, drugs quality is the most important aspect of the inventory system. Recently, Sindhuja and Arathi (2023) proposed an inventory model for deteriorating products under preservation technology with time-dependent quality demand. The model analyses the effect of preservation technology in a dairy company, in particular Ice Cream Company. A deterministic demand

pattern which is dependent on the product quality was considered. In market reality, the demand of newly introduced product remains constant for some time and after that, when the product becomes popular (through advertisement and customers' trial) the demand rises with time. Singh et.al. (2017) proposes an optimal policy for deteriorating items with time-proportional deteriorating rate and constant and time-dependent linear demand rate. Shortages are not allowed. The model obtained the optimal cycle time and optimal order quantity that minimizes the total cost. Bello and Baraya (2019) extended Singh et.al. (2017) to allow shortages.

This research looks at two-phase demand rates (constant demand rate when the items are newly launched in the market and linear time- dependent demand when the products become popular to customers as a result of advertisement and identification of its quality) and effect of preservation technology investment on the deterioration of pharmaceutical inventory items. The model is developed with objective of minimizing total inventory cost. The inventory items are considered to be delayed deteriorating pharmaceutical products with reduced constant deteriorating rate, constant holding cost and preservation technique involved in the system. A methodology to obtain the optimal decision variables that will minimize the total inventory cost is proposed.

### 3. Research gap and Contribution

The present research looks into both Mahata and Debnath (2022) and Priyamvada and Jaggi (2022) models and precisely extends Sindhuja and Arathi (2023) model to pharmaceutical inventory system in healthcare environment by incorporating realistic features in pharmaceutical settings as follows:

- i) The inventory items are considered to be delayed deteriorating items. That is, deterioration of the products (drugs and pharmaceuticals) does not start immediately on production of the items (as in Sindhuja and Arathi (2023)) neither do they start deteriorating from the supplier's warehouse as in the case of Mahata and Debnath (2022). Even in the retailer's warehouse, the deteriorating process begins after certain period in stock.
- ii) Deterministic two-phase demand (constant and time dependent pattern) is considered. We define the reduced deterioration rate under the preservation technology investment (with some results from Bardhan et al, 2017) by the function:

$$\omega = (1 - m(\xi))\theta \quad (1)$$

where  $m(\xi) = 1 - e^{-\gamma\xi}$ ,  $\gamma$  is the sensitivity parameter of investment to the reduced deteriorating rate and  $\xi$  is the retailer's capital investment in preservation technology per unit time. Under this assumption, if  $\theta$  is the initial deteriorating rate of the product without preservation technology and  $\omega$  is the reduced product deterioration rate under preservation technology investment, then  $\omega \ll \theta$ .

## 4. Model Description and Formulation

### 4.1 Assumptions

The model is developed based on the following assumptions:

- i) The inventory system comprises of only one type of items (Pharmaceutical drugs).
- ii) There is no deterioration in the first phase of the inventory cycle and the deterioration rate,  $\theta$  is constant. During the second phase of the planning horizon, the rate of deterioration (which affect the efficacy of the item) can be reduced due to preservation technology investment.
- iii) The demand rate has two components for the time horizon: constant in the first phase of the cycle and linear function of time in the second phase.
- iv) The replenishment is instantaneous, that is, lead time is zero.
- v) Shortages are not allowed.
- vi) The Planning horizon is infinite.

- vii) There is no replacement or repair of deteriorated units during the cycle period under consideration.
- viii)  $m(\xi)$  is the proportion of reduced deteriorating rate. It is continuous, concave and increasing function of the retailer's capital investment in the preservation technology,  $\xi$ . From Shah et al. (2021),  $m(0) = 0$  and  $\lim_{\xi \rightarrow \infty} m(\xi) = 1$ . It is realistic to assume that,  $m'(\xi) > 0$  to encourage the retailer to invest money in the preservation and also assume that  $m''(\xi) < 0$  to ensure diminishing return in the investment.
- ix) From Mahata and Debnath (2022) the retailer cannot invest most of the money in preservation technology then there is an upper bound in the retailer's capital investment, that is,  $0 < \xi \leq \xi_*$ , where  $\xi_*$  is the maximum amount of money to invest in preservation technology. So the optimal preservation technology investment may be  $\xi \in (0, \xi_*]$ .

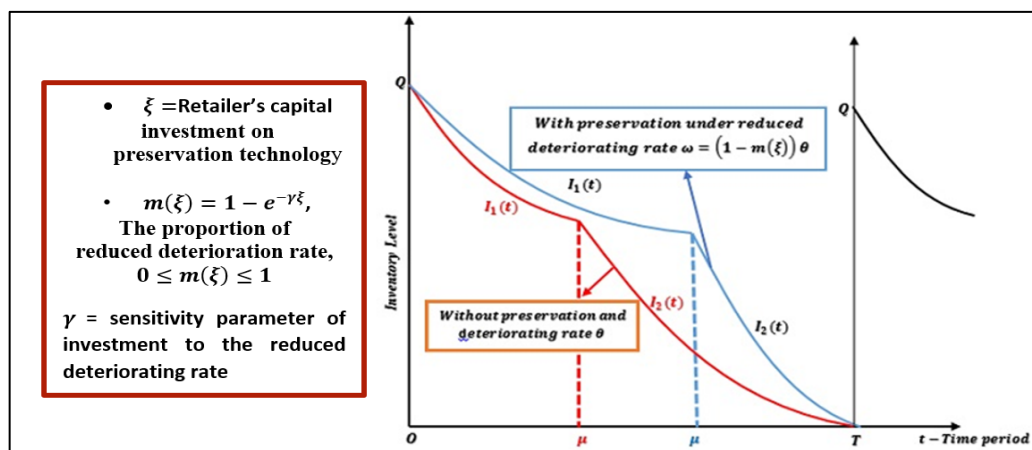
### 4.2 Notations

The following notations are used throughout the paper:

- $A$  Fixed ordering cost in Naira per order.
- $c$  Unit cost price in Naira per unit item.
- $h$  Holding cost in Naira per unit per unit time.
- $H_c$  Total inventory holding cost per cycle.
- $\mu$  Length of time in which the product has no deterioration.
- $\omega$  Reduced deteriorating rate due to preservation technology investment.
- $\xi$  Retailer's capital investment on preservation technology.
- $m(\xi)$  The proportion of reduced deteriorating rate.
- $\xi_*$  Maximum retailer's capital investment on preservation technology.
- $I_1(t)$  Inventory level in free deterioration period,  $[0, \mu]$ .
- $I_2(t)$  Inventory level during deterioration period,  $[\mu, T]$ .
- $\theta$  Deteriorating rate without preservation technology investment.
- $Q$  Total ordering quantity.
- $P_c$  Preservation cost in a cycle.
- $T$  Time for which the inventory level drops to zero and the cycle period (**Decision variable**).
- $TVC(T)$  Average inventory cost per unit time (**Objective function**).

### 4.3 Model Formulation

The inventory cycle starts with the initial inventory  $Q$  at  $t = 0$ . During the interval  $[0, \mu]$ , the inventory level decreases due to the effect of constant demand  $a$  units per unit time. During the interval  $[\mu, T]$  the inventory depletes due to the combined effect of demand (linear time dependent) and reduced deterioration rate due to the introduction of preservation technology investment and finally drops to zero level at  $t = T$ . The behavior of the inventory system is shown diagrammatically in Figure 1.



**Figure 1:** Delayed deteriorating pharmaceutical Inventory system with and without preservation technology investment.

The behavior of the inventory system can be represented by the differential equations.

$$\frac{dI_1(t)}{dt} = -a, \quad 0 \leq t \leq \mu. \tag{2}$$

$$\frac{dI_2(t)}{dt} + \omega I_2(t) = -[a + b(t - \mu)], \quad \mu \leq t \leq T \tag{3}$$

Equation (1) is first order variable separable differential equation with solution,

$$I_1(t) = -at + c_1. \tag{4}$$

where  $c_1$  is arbitrary constant. Putting the boundary condition  $I_1(0) = Q$  in equation (4), we get,  $c_1 = Q$ . Equation (4) becomes,

$$I_1(t) = Q - at. \tag{5}$$

Equation (3) is first order linear differential equation with integrating factor,  $\rho = e^{\int \omega dt}$ .

$$\Rightarrow \rho = e^{\omega t}. \tag{6}$$

The solution of equation (3) is obtained as follows:

$$\begin{aligned} e^{\omega t} I_2(t) &= - \int e^{\omega t} [a + b(t - \mu)] dt \\ &= -a \int e^{\omega t} dt - b \int t e^{\omega t} dt + b\mu \int e^{\omega t} dt \\ &= -\frac{a}{\omega} e^{\omega t} - b \left( \frac{t}{\omega} e^{\omega t} - \frac{1}{\omega^2} e^{\omega t} \right) + \frac{b\mu}{\omega} e^{\omega t} + c_2, \text{ where } c_2 \text{ is integrating constant.} \\ \therefore I_2(t) &= -\frac{a}{\omega} - b \left( \frac{t}{\omega} - \frac{1}{\omega^2} \right) + \frac{b\mu}{\omega} + c_2 e^{-\omega t}. \end{aligned} \tag{7}$$

Putting the boundary condition  $I_2(T) = 0$  in equation (7), we get,

$$\begin{aligned} c_2 e^{-\omega T} &= \frac{a}{\omega} + b \left( \frac{T}{\omega} - \frac{1}{\omega^2} \right) - \frac{b\mu}{\omega} \\ \Rightarrow c_2 &= \frac{a}{\omega} e^{\omega T} - b \left( \frac{T}{\omega} - \frac{1}{\omega^2} \right) e^{\omega T} + \frac{b\mu}{\omega} e^{\omega T}. \end{aligned} \tag{8}$$

Substituting Equation (8) into Equation (7) and simplifying we obtain the final expression for  $I_2(t)$  as:

$$I_2(t) = \frac{(1 - e^{\omega(T-t)})}{\omega} \left[ \frac{b}{\omega} + b\mu - a \right] + \frac{b}{\omega} (T e^{\omega(T-t)} - t). \tag{9}$$

By the continuity of  $I(t)$  at  $t = \mu$ , we have  $I_1(\mu) = I_2(\mu)$ . Thus, we obtain from equations (5) and (9),

$$Q - a\mu = \frac{(1 - e^{\omega(T-\mu)})}{\omega} \left[ \frac{b}{\omega} + b\mu - a \right] + \frac{b}{\omega} (T e^{\omega(T-\mu)} - \mu).$$

$\therefore$  The order quantity is,

$$Q = a\mu + \frac{(1 - e^{\omega(T-\mu)})}{\omega} \left[ \frac{b}{\omega} + b\mu - a \right] + \frac{b}{\omega} (T e^{\omega(T-\mu)} - \mu). \tag{10}$$

Substituting equation (10) into equation (5) we obtain the final expression of  $I_1(t)$  as:

$$I_1(t) = \frac{(1-e^{\omega(T-\mu)})}{\omega} \left[ \frac{b}{\omega} + b\mu - a \right] + \frac{b}{\omega} (Te^{\omega(T-\mu)} - \mu) + a(\mu - t). \tag{11}$$

The cost function of pharmaceutical inventory system consists of the following cost components:

i) Fixed ordering cost,  $OC = A$ .

ii) Inventory Holding Cost,  $H_c = h \int_0^T I(t)dt = h \int_0^\mu I_1(t)dt + h \int_\mu^T I_2(t)dt$

$$\begin{aligned} &= h \int_0^\mu \left\{ \frac{(1-e^{\omega(T-\mu)})}{\omega} \left[ \frac{b}{\omega} + b\mu - a \right] + \frac{b}{\omega} (Te^{\omega(T-\mu)} - \mu) + a(\mu - t) \right\} dt \\ &\quad + h \int_\mu^T \left\{ \frac{(1-e^{\omega(T-t)})}{\omega} \left[ \frac{b}{\omega} + b\mu - a \right] + \frac{b}{\omega} (Te^{\omega(T-t)} - t) \right\} dt. \\ &= h \left\{ \begin{aligned} &\frac{b}{\omega^3} (1 - e^{\omega(T-\mu)}) + \frac{a}{\omega^2} (e^{\omega(T-\mu)} - 1) + \frac{a\mu}{\omega} \left( e^{\omega(T-\mu)} + \frac{\mu\omega}{2} \right) - \frac{b\mu^2}{\omega} (e^{\omega(T-\mu)} + 1) \\ &+ \frac{b\mu}{\omega^2} (1 - 2e^{\omega(T-\mu)}) + \frac{b\mu}{\omega} (e^{\omega(T-\mu)} + 1)T + \frac{b}{\omega^2} Te^{\omega(T-\mu)} - \frac{a}{\omega} T + \frac{b\mu^2}{2\omega} - \frac{b}{2\omega} T^2 \end{aligned} \right\}. \end{aligned} \tag{12}$$

iii) Deteriorating cost,  $D_c = c\omega \int_\mu^T I_2(t)dt$

$$\begin{aligned} &= c\omega \int_\mu^T \left[ \frac{(1 - e^{\omega(T-t)})}{\omega} \left( \frac{b}{\omega} + b\mu - a \right) + \frac{b}{\omega} (Te^{\omega(T-t)} - t) \right] dt \\ &= c\omega \left[ \begin{aligned} &\frac{b}{\omega^3} (1 - e^{\omega(T-\mu)}) + \frac{a}{\omega^2} (e^{\omega(T-\mu)} - 1) + \frac{bT}{\omega^2} \left( e^{\omega(T-\mu)} - \frac{\omega}{2} T \right) - \frac{b\mu}{\omega^2} e^{\omega(T-\mu)} \\ &-\frac{a}{\omega} (T - \mu) + \frac{b\mu}{\omega} T + \frac{b\mu^2}{2\omega} - \frac{b\mu^2}{\omega} \end{aligned} \right]. \end{aligned} \tag{13}$$

iv) Preservation technology cost is given by:

$$PT_c = \xi \times T = \xi T. \tag{14}$$

The total cost of the inventory system per unit cycle time is given by:

$$\begin{aligned} TC(T) &= \frac{1}{T} (A + H_c + D_c + PT_c) \\ TVC(T) &= \frac{1}{T} \left\{ \begin{aligned} &A + h \left[ \frac{b}{\omega^3} (1 - e^{\omega(T-\mu)}) + \frac{a}{\omega^2} (e^{\omega(T-\mu)} - 1) + \frac{a\mu}{\omega} \left( e^{\omega(T-\mu)} + \frac{\mu\omega}{2} \right) - \frac{b\mu^2}{\omega} (e^{\omega(T-\mu)} + 1) \right] \\ &+ \frac{b\mu}{\omega^2} (1 - 2e^{\omega(T-\mu)}) + \frac{b\mu}{\omega} (e^{\omega(T-\mu)} + 1)T + \frac{b}{\omega^2} Te^{\omega(T-\mu)} - \frac{a}{\omega} T + \frac{b\mu^2}{2\omega} - \frac{b}{2\omega} T^2 \\ &+ c\omega \left[ \frac{b}{\omega^3} (1 - e^{\omega(T-\mu)}) + \frac{a}{\omega^2} (e^{\omega(T-\mu)} - 1) + \frac{bT}{\omega^2} \left( e^{\omega(T-\mu)} - \frac{\omega}{2} T \right) - \frac{b\mu}{\omega^2} e^{\omega(T-\mu)} \right] \\ &-\frac{a}{\omega} (T - \mu) + \frac{b\mu}{\omega} T + \frac{b\mu^2}{2\omega} - \frac{b\mu^2}{\omega} \end{aligned} \right\} + \xi T. \end{aligned} \tag{15}$$

### 5. Optimality Analysis

The necessary and sufficient conditions for the optimality of the objective cost function are respectively,

$$\frac{dTVC(T)}{dT} = 0, \text{ and } \frac{d^2TVC(T)}{dT^2} > 0.$$

$$\begin{aligned} \frac{dTVC(T)}{dT} = & -\frac{A}{T^2} - \frac{bh}{T^2\omega^3} - \frac{bh(T\omega e^{\omega(T-\mu)} - e^{\omega(T-\mu)})}{\omega^3 T^2} + \frac{ah(T\omega e^{\omega(T-\mu)} - e^{\omega(T-\mu)})}{\omega^2 T^2} + \frac{ah}{\omega^2 T^2} \\ & + \frac{a\mu h(T\omega e^{\omega(T-\mu)} - e^{\omega(T-\mu)})}{\omega T^2} \\ & + \frac{a\mu^2 h(T\omega e^{\omega(T-\mu)} - e^{\omega(T-\mu)})}{2 T^2} - \frac{b\mu^2 h(T\omega e^{\omega(T-\mu)} - e^{\omega(T-\mu)})}{\omega T^2} + \frac{b\mu^2 h}{\omega T^2} - \frac{b\mu h}{\omega^2 T^2} - \frac{2b\mu h(T\omega e^{\omega(T-\mu)} - e^{\omega(T-\mu)})}{\omega^2 T^2} \\ & + b\mu h e^{\omega(T-\mu)} + \frac{bh}{\omega} e^{\omega(T-\mu)} - \frac{bh}{2\omega} - \frac{b\mu^2 h}{2\omega T^2} - \frac{bc}{\omega^2 T^2} - \frac{bc(T\omega e^{\omega(T-\mu)} - e^{\omega(T-\mu)})}{\omega^2 T^2} + \frac{ac(T\omega e^{\omega(T-\mu)} - e^{\omega(T-\mu)})}{\omega T^2} \\ & + \frac{ac}{\omega T^2} + bce^{\omega(T-\mu)} - \frac{bc}{2} - \frac{bc\mu(T\omega e^{\omega(T-\mu)} - e^{\omega(T-\mu)})}{\omega T^2} - \frac{ac\mu}{T^2} - \frac{c\mu^2}{2T^2}. \end{aligned} \tag{16}$$

$$\begin{aligned} \frac{d^2TVC(T)}{dT^2} = & \frac{2A}{T^3} + \frac{2bh}{\omega^3 T^3} + \frac{bh(T^2\omega e^{\omega(T-\mu)} - 2Te^{\omega(T-\mu)})}{\omega^3 T^4} - \frac{bh(T\omega e^{\omega(T-\mu)} - e^{\omega(T-\mu)})}{\omega^2 T^2} \\ & - \frac{ah(T^2\omega e^{\omega(T-\mu)} - 2Te^{\omega(T-\mu)})}{\omega^2 T^4} + \frac{ah(T\omega e^{\omega(T-\mu)} - e^{\omega(T-\mu)})}{\omega T^2} - \frac{2ah}{\omega^2 T^3} - \frac{a\mu h(T^2\omega e^{\omega(T-\mu)} - 2Te^{\omega(T-\mu)})}{\omega T^4} + \frac{a\mu h(T\omega e^{\omega(T-\mu)} - e^{\omega(T-\mu)})}{\omega T^2} \\ & - \frac{a\mu^2 h(T^2\omega e^{\omega(T-\mu)} - 2Te^{\omega(T-\mu)})}{2 T^4} + \frac{a\mu^2 \omega h(T\omega e^{\omega(T-\mu)} - e^{\omega(T-\mu)})}{2 T^2} + \frac{b\mu^2 h(T^2\omega e^{\omega(T-\mu)} - 2Te^{\omega(T-\mu)})}{\omega^2 T^4} \\ & - b\mu^2 h \frac{(T\omega e^{\omega(T-\mu)} - e^{\omega(T-\mu)})}{T^2} - \frac{2b\mu^2 h}{\omega T^3} + \frac{2b\mu h}{\omega^2 T^3} + \frac{2b\mu h(T^2\omega e^{\omega(T-\mu)} - 2Te^{\omega(T-\mu)})}{\omega^2 T^4} - \frac{2b\mu h(T\omega e^{\omega(T-\mu)} - e^{\omega(T-\mu)})}{\omega T^2} \\ & + b\mu h \omega e^{\omega(T-\mu)} + bhe^{\omega(T-\mu)} + \frac{b\mu^2 h}{\omega T^3} + \frac{2bc}{\omega^2 T^3} + \frac{bc(T^2\omega e^{\omega(T-\mu)} - 2Te^{\omega(T-\mu)})}{\omega^2 T^4} - \frac{bc(T\omega e^{\omega(T-\mu)} - e^{\omega(T-\mu)})}{\omega T^2} \\ & - \frac{ac(T^2\omega e^{\omega(T-\mu)} - 2Te^{\omega(T-\mu)})}{\omega T^4} + ac \frac{(T\omega e^{\omega(T-\mu)} - e^{\omega(T-\mu)})}{T^2} - \frac{2ac}{\omega T^3} + bc\omega e^{\omega(T-\mu)} \\ & + \frac{b\mu c(T^2\omega e^{\omega(T-\mu)} - 2Te^{\omega(T-\mu)})}{\omega T^4} \\ & - bc \frac{(T\omega e^{\omega(T-\mu)} - e^{\omega(T-\mu)})}{T^2} + \frac{2ac\mu}{T^3} + \frac{c\mu^2}{T^3}. \end{aligned} \tag{17}$$

Due to the complexity of the second derivative function in equation (17), it is difficult to carry out the convexity test analytically. We numerically show that the cost function is convex in T by showing that  $\frac{d^2TVC(T)}{dT^2} > 0$ .

The objective of the model is to find the optimal replenishment policy that minimizes the retailer's total cost at reduced deteriorating rate due to investment in preservation technology. To obtain the optimal value of the cycle period T, we set  $\frac{dTVC(T)}{dT} = 0$ . From equation (16) we get:

$$\begin{aligned} -A - \frac{bh}{\omega^3} + \frac{bh}{\omega^3} e^{\omega(T-\mu)} - \frac{bh}{\omega^2} T e^{\omega(T-\mu)} - \frac{ah}{\omega^2} e^{\omega(T-\mu)} + \frac{ah}{\omega} T e^{\omega(T-\mu)} + \frac{ah}{\omega^2} - \frac{a\mu h}{\omega} e^{\omega(T-\mu)} + a\mu h T e^{\omega(T-\mu)} \\ - \frac{a\mu^2 h}{2} e^{\omega(T-\mu)} + \frac{a\mu^2 h \omega}{2} T e^{\omega(T-\mu)} + \frac{b\mu^2 h}{\omega} e^{\omega(T-\mu)} - b\mu^2 h T e^{\omega(T-\mu)} + \frac{b\mu^2 h}{\omega} - \frac{b\mu h}{\omega^2} + \frac{2b\mu h}{\omega^2} e^{\omega(T-\mu)} \\ - 2b\mu h T e^{\omega(T-\mu)} + b\mu h T^2 e^{\omega(T-\mu)} + \frac{bh}{\omega} T^2 e^{\omega(T-\mu)} - \frac{bh}{2\omega} T^2 - \frac{b\mu^2 h}{2\omega} - \frac{bc}{\omega^2} + \frac{bc}{\omega^2} e^{\omega(T-\mu)} - \frac{bc}{\omega} T e^{\omega(T-\mu)} \\ - \frac{ac}{\omega} e^{\omega(T-\mu)} + ac T e^{\omega(T-\mu)} + \frac{ac}{\omega} + bc T^2 e^{\omega(T-\mu)} - \frac{bc}{2} T^2 + \frac{bc\mu}{\omega} e^{\omega(T-\mu)} - bc T e^{\omega(T-\mu)} - ac\mu - \frac{c\mu^2}{2} = 0. \end{aligned} \tag{18}$$



## 6. Solution Algorithm

In order to obtain the optimal solution of the inventory system, we propose the following algorithm:

**Step I:** Start

**Step II:** Input the appropriate values of the system parameters.

**Step III:** Using Maple 18 Mathematical software solve equation (18),  $\frac{dTC(T)}{dT} = 0$  and find T.

**Step IV:** Compare the obtained value of T from step III with  $\mu$ .

(i) If  $T > \mu$  then T is feasible solution. Evaluate  $\frac{d^2TC(T)}{dT^2} > 0$ , the value of  $T = T^*$  is the optimal value.

Then go to step V.

(ii) If  $T < \mu$  then T is infeasible. Choose another set of parameters and repeat step III to IV.

**Step V:** Substitute  $T = T^*$  in equations (10) and (15) to obtain the corresponding optimal order quantity,  $Q^*$  and optimal total cost  $TVC(T^*)$  respectively.

**Step VI:** End.

## 7. Numerical Examples

Consider the pharmaceutical inventory system with the following input parameters:

$$\theta = 0.2, \gamma = 0.01, A = 200, a = 500, b = 0.2, c = 250, h = 5, \mu = 0.4, \xi = 100,$$

$$\omega = 1 - e^{-\gamma\xi} = 0.0736.$$

The optimal cycle time,  $T^* = 1.3356$  years (487 days), optimal ordering quantity,  $Q^* = 719$  units, the minimum total cost per unit time,  $TVC(T^*) = 267.27$  and the sufficient condition is,  $\frac{d^2TVC(T)}{dT^2} = 6.78 \times 10^6 > 0$ .

**Table 1.** Effect of preservation investment on deterioration rate, cycle length, inventory cost and reorder quantity

$\xi$	$\omega$	$T^*$	$TVC(T^*)$	$Q^*$
100	0.0736	1.3356	267.27	719
110	0.0666	1.3416	275.21	717
120	0.0602	1.3470	283.36	715
130	0.0545	1.3519	291.72	714
140	0.0493	1.3564	300.22	712
150	0.0446	1.3604	308.89	711
160	0.0438	1.3611	318.66	710
170	0.0365	1.3674	326.59	709
180	0.0331	1.3704	335.63	708
190	0.0299	1.3732	344.73	707
200	0.0271	1.3756	353.95	706

**NOTE:** Increase in  $\xi$  produces corresponding decrease in  $\omega$  and  $Q^*$ , with increase in  $T^*$  and  $TVC(T^*)$ .

## 8. Sensitivity Analysis

In this section we study the effect of changes in the values of some sensitive system parameters  $a, b, \omega, c, h$  and  $\mu$  on the optimum cost, cycle period and optimum order quantity. The sensitivity analysis is carried out by changing each parameter by +25, +15, +5, 0, -25, -15 and -5%, taking one parameter at a time and keeping the remaining parameters constant. The analysis is based on the numerical example 1. The results are depicted in Table 2.

**Table 2.** Effect of changes of the parameter values on optimal solution

Parameter	% change in parameter values	$T^*$	$TVC(T^*)$	% change in $TVC(T^*)$	$Q^*$	% change in $Q^*$
<b>a</b>	+25	1.3354	267.2907	+0.0094	898.4909	+25.0028
	+15	1.3355	267.2782	+0.0047	826.6052	+15.0017
	+5	1.3356	267.2656	+0.0000	754.7195	+5.0006
	<b>0</b>	<b>1.3356</b>	<b>267.2656</b>	<b>0.0000</b>	<b>718.7765</b>	<b>0.0000</b>
	-5	1.3357	267.2531	-0.0047	682.8338	-5.0005
	-15	1.3358	267.2406	-0.0094	610.9481	-15.0017
	-25	1.3360	267.2156	-0.0187	539.0627	-25.0027
<b>b</b>	+25	1.3356	267.2656	+0.0000	718.7557	-0.0029
	+15	1.3356	267.2656	+0.0000	718.7640	-0.0017
	+5	1.3356	267.2656	+0.0000	718.7724	-0.0006
	<b>0</b>	<b>1.3356</b>	<b>267.2656</b>	<b>0.0000</b>	<b>718.7765</b>	<b>0.0000</b>
	-5	1.3356	267.2656	-0.0000	718.7807	+0.0006
	-15	1.3356	267.2656	-0.0000	718.7890	+0.0017
	-25	1.3356	267.2656	-0.0000	718.7974	+0.0029
<b>ω</b>	+25	1.3202	272.7011	+2.0337	723.6380	+0.6764
	+15	1.3263	270.5120	+1.2147	721.6755	+0.4033
	+5	1.3359	267.9205	+0.2450	719.7588	+0.1367
	<b>0</b>	<b>1.3356</b>	<b>267.2656</b>	<b>0.0000</b>	<b>718.7765</b>	<b>0.0000</b>
	-5	1.3388	266.1749	-0.4081	717.8063	-0.1350
	-15	1.3450	264.0520	-1.2024	715.8989	-0.4003
	-25	1.3513	261.9182	-2.0008	713.9748	-0.6680
<b>c</b>	+25	1.3356	270.6970	+1.2839	718.7768	+0.0000
	+15	1.3356	269.3321	+0.7732	718.7765	+0.0000
	+5	1.3356	267.9545	+0.2578	718.7765	+0.0000
	<b>0</b>	<b>1.3356</b>	<b>267.2656</b>	<b>0.0000</b>	<b>718.7765</b>	<b>0.0000</b>
	-5	1.3356	266.5768	-0.2577	718.7765	-0.0000
	-15	1.3356	265.1992	-0.7732	718.7765	-0.0000
	-25	1.3356	263.8215	-1.2886	718.7765	-0.0000
<b>h</b>	+25	1.3353	268.2393	+0.3643	718.7756	-0.0001
	+15	1.3354	267.8523	+0.2195	718.7759	-0.00008
	+5	1.3355	267.4654	+0.0748	718.7762	-0.00004
	<b>0</b>	<b>1.3356</b>	<b>267.2656</b>	<b>0.0000</b>	<b>718.7765</b>	<b>0.0000</b>
	-5	1.3357	267.0660	-0.0747	718.7768	+0.00004
	-15	1.3358	266.6791	-0.2194	718.7771	+0.00008
	-25	1.3360	266.2799	-0.3688	718.7777	+0.0002
<b>μ</b>	+25	1.4278	256.4645	-4.0413	768.7537	+6.9531
	+15	1.3910	260.6039	-2.4925	748.7631	+4.1719
	+5	1.3541	264.9804	-0.8550	728.7721	+1.3894
	<b>0</b>	<b>1.3356</b>	<b>267.2656</b>	<b>0.0000</b>	<b>718.7765</b>	<b>0.0000</b>
	-5	1.3172	269.6022	+0.8743	708.7812	-0.0139
	-15	1.2802	274.5034	+2.7081	688.7900	-4.1719
	-25	1.2433	279.6831	+4.6461	668.7990	-6.9531

**9. Observations on Sensitivity Analysis**

- i) When the value of  $a$  (the constant demand) increases, corresponding increase in average total cost per unit time and optimal order quantity with decrease in cycle period are observed. Therefore, it is advisable for the pharmacy stock manager to increase the order quantity when demand increases in order to gain more profit at optimal cost.
- ii) When the value of the parameter  $b$  increases, both the cycle period and average cost remain unchanged with corresponding decrease in optimal order quantity. This indicated that, increase in the varying demand coefficient required the reduction of the order quantity in order to optimize the average cost.
- iii) When the value of  $\omega$  (the reduced deterioration rate due to the investment in preservation technology) increases, it is obvious that the average cost increases as more capital is introduced into the investment. Also, corresponding increase in order quantity and decrease in cycle length are observed. Therefore, it is advisable for the stock manager to increase the order quantity since more items will be bought due to their freshness as a result of preservation investment.
- iv) When the value of  $c$  (the unit cost of item) increases, both the cycle length and order quantity remain unchanged. But corresponding increase in average cost is observed.
- v) When the value of  $h$  (unit holding cost of item) increases, both the cycle length and order quantity remain unchanged. But corresponding increase in average cost is observed.
- vi) When the value of the parameter  $\mu$  (free deterioration period) increases, there is corresponding increase in both the optimal cycle length,  $T^*$  and optimal order quantity,  $Q^*$ , with decrease in the average cost. This indicates that the pharmacy manager needs to increase the order quantity when  $\mu$  increases in order to minimize the cost.

## 10. Conclusion

The research presents an inventory ordering policy which is applicable to pharmaceutical enterprises in healthcare environment. The model developed considers delayed deteriorating pharmaceutical items (drugs). Two stages of inventory cycle are considered: free deteriorating stage (where the depletion of the inventory depends on the constant demand only) and the period when deterioration commences (where the depletion of the inventory depends on both the linear-time dependent demand and reduced deteriorating rates due to the investment in preservation technology). This kind of two-phase demand inventory system is seen in the situation when new items (drugs) are introduced. The demand stays constant in the early stage, and shortly after, when the products become popular in the pharmacy or market, on the identification of the quality of the product and through advertisement, the demand rises. The optimal order quantity and optimal cycle period have been derived by minimizing the total inventory cost per unit time. Maple 18 Mathematical software is used to solve the problem. Validity of the model is tested with several numerical examples and followed by sensitivity analysis. The observation on the analysis helps to draw positive managerial decisions for the stock manager. The proposed model can further be extended with stochastic deteriorating rates, product quality time dependent demand, stochastic demand pattern, time dependent ordering cost and permissible delay in payment.

## References

- Bardhan, S., Pal, H. and Giri, B.C. (2017). Optimal replenishment policy and preservation technology investment for non-instantaneous deteriorating item with stock-dependent demand, Springer, *Operational Research International Journal*, pp. 1-21.
- Bello, Y., Baraya, Y. M. (2019). An Inventory Model for Non-instantaneous Deteriorating Items with Two-phase demand Rates and Exponential Backlogging, *Abacus (Mathematical Science Series)*, 44(1), pp. 377-390.
- Chang, H.J, Dye, C.Y. (1999). An EOQ model for deteriorating items with time varying demand and partial backlogging, *Journal of Operational Research Society* 50, pp. 1176–1182
- Chung K.J, Ting P.S. (1993). A heuristic for replenishment for deteriorating items with a linear trend in demand. *Journal of Operational Research Society* 44, pp. 1235–1241

- Combin A., Martein, L. (2009). Generalized Convexity and Optimization: Theory and applications: published by Springer-Verlag Berlin Heidelberg, p33.
- Dave U, Patel L.K (1981). Inventory policy inventory model for deteriorating items with time proportional demand, *Journal of Operational Research Society* 32, pp. 137–142.
- De, L.N., Mallik, P. (2020). Effect of Price Concession Strategy and Preservation Technology on Inventory Model for Decaying Products with Partial Backlogging and Price-Stock-Dependent Demand, *International Journal of Engineering Research and Technology*, 13(6), pp. 1420-1431.
- Dye, C., (2013). The effect of preservation technology investment on non-instantaneous deteriorating inventory model, *Elsevier, Omega: The international Journal of Management Science*, (41), 872-880
- Ghare, P. M., Schrader, G.P. (1963). A Model for Exponentially Decaying Inventory, *Journal of Industrial Engineering*, 14(5), pp. 238-243.
- Hishamuddin, I., Supadi, S.S. and Omar, M. (2020). Improved Preservation Technology for Non- Instantaneous Deteriorating Inventory Using Boundary Condition Estimation, *Applied and Computational Mathematics*, 9(4), pp. 118-129.
- Hsu, P.H., Wee, H.M. and Teng, H.M. (2010). Preservation technology investment for deteriorating inventory, Elsevier, *International Journal of Production Economics*, 124, pp. 388-394.
- Huang, Y., Wang, C., Huang, C. and Dye, C. (2011). Comments on preservation technology investment for deteriorating inventory, *African Business Management*, 5(11), pp. 4636-4643.
- Kumar, V., Sharma, A., and Gupta C.V. (2014). An EOQ model for time-dependent demand and parabolic holding cost with preservation technology under partial backlogging for deteriorating items, *International of Education and Science Research Review*, 1(2), pp. 179-185.
- Mahata, S., Debnath, B.K. (2022). A profit maximization inventory problem considering deterioration during carrying for price dependent and preservation technology investment, *RAIRO Operations Research*, 56, pp.1841-1856.
- Mishra, P., Shaik, A. (2017). Optimal Policies for Deteriorating Items with Preservation and Maintenance Management when Demand is Trade Credit Sensitive, *AMSE Journals - AMSE IIETA publication -2017 Series: Modelling D*, 38, pp. 36-54.
- Mishra, V.K. (2013). Deteriorating inventory model using preservation technology with salvage value and shortages, *Advances in Production Engineering and Management (APEM)*, 8(3), pp. 1854-6250.
- Priyamvada, R., Jaggi, C.K. (2022). Optimal strategies for deteriorating items with Price- sensitive investment in preservation technology, *RAIRO Operations Research*, 56, pp. 601-617.
- Shah, N.H., Shah, P.H. and Patel, M.B, (2021). Retailer’s Inventory Decisions for Promotional efforts and Preservation Technology Investments when Supplier offers Quantity Discounts, *OPSEARH*, pp.1-17.
- Shah, Y.K., (1977). An order-level lot-size inventory model for deteriorating items. *AIIE Transactions*, (91), pp. 542-582.
- Sindhuja, S. Arathi, P. (2023). An inventory model for deteriorating products under preservation technology with time-dependent quality demand, *Cogent Engineering*, 10, pp. 1-25.
- Singh, S.R., Rathore, H. (2015). Optimal Payment Policy with Preservation Technology Investment and Shortages under Trade Credit, *Indian Journal of Science and Technology*, 8(S7), pp. 203-212.
- Singh, T., Mishra, P.J. and Pattanayak, H. (2017). An Optimal Policy for deteriorating Items with Time proportional Demand and Constant and Time-dependent Linear Demand, *International Journal of Industrial Engineering*, pp. 1-9.
- Wee, H.M. (1995). A deterministic lot-size inventory model for deteriorating items with shortages and a declining market. *Computational Operation research* 22, pp. 345–356.
- Zhang, J., Bai, Z., Tang, W. (2014). An optimal pricing policy for deteriorating items with preservation technology investment, *Journal of Industrial and management Optimization*, 10(4), pp. 1261-1277.