

# Mathematical Modelling of MHD Couette flow of Burger's fluid in annuli

Sani Isa\* and Ali Musa

Department of Mathematics and Statistics, Yobe State University Damaturu, Nigeria

\*Corresponding author emails: [sanimath@yahoo.com](mailto:sanimath@yahoo.com); [kukuwaalimusa@gmail.com](mailto:kukuwaalimusa@gmail.com)  
<https://orcid.org/0000-0002-6938-5354>; <https://orcid.org/0009-0002-1376-3317>

## Abstract

*This paper presents a semi-analytical study for the MHD Couette flow of Burger's fluid. The flow of an incompressible conducting Burgers fluid in the presence of a uniform transverse magnetic field between two concentric cylinders that is moved suddenly is considered. By the application of Laplace Transformation (LT) and Riemann-sum approximation method (RSM), the semi-analytical expressions for velocity and the skin friction are obtained. The influence of various dimensionless parameters emerging in the model such as time ( $t$ ), magnetic parameter ( $M$ ), relaxation ( $\lambda$ ), retardation ( $\theta$ ) times and Burger's ( $\beta$ ) parameter on the velocity and skin friction are presented graphically and discussed. It is found that the relaxation time helps the flow pattern, while material constants resist the fluid flow. Finally, earlier solutions to the Oldroyd-B fluid, Maxwell fluid and Newtonian fluid appears as the limiting case of the present solution.*

**Keywords:** Burger's fluid, MHD, Couette flow, Annuli, Riemann-sum approximation, Rheometer

## 1. Introduction

Studies of unsteady MHD flows has gained increasing interest from the practical point of view due its application in many natural phenomena and engineering problems. This is due to the fact that fluid transient may be expected at the start up time of MHD devices such as, MHD power generators and pumps, MHD accelerators, nuclear reactors and so forth. The first study of MHD flow of non-Newtonian fluids was carried out by Sarpkaya (1961). Since then, several other investigations considering the MHD flow of non-Newtonian fluids were carried out and as such it becomes an active area of research. Ersoy (1999) examined the MHD flow between eccentric rotating disks for an Oldroyd-B fluid. Liu, et al. (2011) and Zheng et al. (2011) and (2012) analyzed the MHD flow of generalized Oldroyd-B fluid for different physical situations using fractional derivatives. Hayat et al. (2004) considered the motion of electrically conducting incompressible Oldroyd-B fluid between two infinitely extended non-conducting parallel plates under a uniform transverse magnetic field, fixed relative to the fluid. Their results showed that the flow field is appreciably influenced by the applied magnetic field, the rotation and the material parameters of the fluid. An exact solution for flows of an electrically conducting Oldroyd-B fluid over an infinite oscillatory plate in the presence of a transverse magnetic field was obtained by Hayat and Hutter (2002). Khan et al. (2012) derived exact solutions of Stokes second problem for MHD Oldroyd-B fluid. Sultan et al. (2013) presented analytical solution for the unsteady magnetohydrodynamic (MHD) flow of Oldroyd-B fluid in long porous rectangular cross-section. Recently Khan *et al.* (2013) presented an analytical solution to the problem of MHD flow of an Oldroyd-B fluid induced by the impulsive motion of a plate between two side walls perpendicular to the plate an extension to the work by Fetecau et al. (2011). The analytical solutions are carried out by using the Fourier sine and Laplace transforms. An exact solutions corresponding to the axial flow of an Oldroyd-B fluid in an annular region between two infinite circular cylinders are established by Jamil et al. (2012).

The nonlinearity nature of relationship between shear stress and shear rate that occur due to the variation of viscosity at a given temperature and pressure in non-Newtonian fluid makes it uneasy to have a constitutive relation that predicts their behaviors. This brought about the growing concern in the processing industries and elsewhere where materials whose flow behavior in shearing cannot be characterized by Newtonian relationship. There are many investigations that has been done on the dynamics of non-Newtonian fluids. Hence, of significance to this present study are those on Burger’s fluid whose application are found in the field of Biological and Chemical analysis, drug delivery, bacteria detection, slurries and many others. Extensive applications of Burgers fluid can be found in the literature and we refer readers to the works of Burger (1935), Krishnan and Rajagopal (2004), Chen et al (2004), Ravindran et al (2004), Ravindran (2004), Chun et al. (2006), Chen et al. (2006), Khan et al. (2010), Hyder (2010), Yaqing Liu et. al. (2011), Quintanilla and Rajagopal (2012), Khan (2013), Alsaedi et al (2013), Ahmad et al (2014), Jianhong et al. (2015), Yan Zhang et. al (2017), Kashif et al. (2018), Madeeha et al. (2018), Nirmala and Kumari (2019), Fetecau and Dumitru (2021), Maria et al. (2022), and Musa et al. (2023).

Motivated by the above mentioned works, this paper presents an extension of Sani and Jha, (2022) from an Oldroyd-B fluid to the Burgers’ fluid. The expressions for the velocity and shear stress are given in Laplace transform domain with the aid of modified Bessel equations which fulfill all the imposed conditions. Inversion of the Laplace transformation is done numerically by using MATLAB software. At the end, influence of various parameters like relaxation and retardation time parameters, material constants, time and viscosity parameters are graphically presented.

## 2. Governing Equations

The flow of an unsteady incompressible and electrically conducting Burgers’ fluid filling the gap between two horizontal concentric cylinders of infinite length governed by the following equations are considered.

$$\nabla V = 0. \tag{1}$$

$$\rho \frac{dV}{dt} = -\nabla P + \nabla S + J \times B. \tag{2}$$

The Cauchy stress tensor  $T$  and the extra stress tensor  $S$  of an incompressible Burgers’ fluid defined by the constitutive equation as;

$$T = -pI + S. \tag{3}$$

and

$$S + \lambda \frac{DS}{Dt} + \beta \frac{DS}{Dt} = \mu \left( 1 + \theta \frac{D}{Dt} \right) A_1. \tag{4}$$

Where  $p$  is the pressure,  $I$  is the identity tensor,  $\mu$ ,  $\lambda$ ,  $\theta$ , and  $\beta$  are the material constants and respectively known as the viscosity coefficient, the relaxation time, retardation times and Burger’s parameter. It is assumed that  $\lambda \geq \theta \geq \beta \geq 0$ .  $A_1$  is the first Rivlin–Ericksen tensor and  $\frac{D}{Dt}$  is the upper convected derivative defined by

$$A_1 = \nabla V + (\nabla V)^T. \tag{5}$$

and

$$\frac{DS}{Dt} = \frac{\partial S}{\partial t} + (V \cdot \nabla)S - STV - (\nabla V)^T S. \tag{6}$$

Where  $V$  being the velocity vector,  $\nabla$  is the gradient operator, the superscript  $T$  denotes the transpose operation. In the present problem eq. (6) reduces to

$$\rho \frac{dV}{dt} = \nabla T + \rho b. \tag{7}$$

Where  $\rho$  is the fluid density,  $b$  is the body force field,  $\frac{d}{dt}$  is the material derivative. The velocity field and the stress is defined as;

$$V = u(r, z, t)i, \quad S = s(r, z, t)i. \tag{8}$$

Where  $\hat{i}$  and  $u$  are the unit vectors and velocity in the radial direction of the cylindrical coordinate system respectively.

### 3. Mathematical Formulation and Solution of the Problem

In this problem, we consider flow formation of an unsteady and electrically conducting incompressible Burgers' fluid due to sudden motion of outer cylinder of an annulus. The fluid flow between the two cylinders in the presence of magnetic field acting perpendicular to the flow direction, where  $a$  and  $b$  are the radii of the inner and the outer cylinders respectively. We assume that the magnetic Reynolds number is very small which correspond to negligible induced magnetic field compared to the externally applied one as discussed in Pai, (1962). At time  $t' \leq 0$ , the fluid and the cylinders are assumed to be at rest. However, when  $t' > 0$  the outer cylinder begins to move in its own plane with a velocity  $Ut'^n$ , where  $U$  a constant is and the inner cylinder remains at rest. Since the cylinders are of infinite length and the flow is fully developed, all physical variables are functions of  $r'$  and  $t'$ . Furthermore, assuming that no applied and polarization voltage exists. Under these assumptions, the mathematical model governing the present physical situation is given as;

$$\left(1 + \lambda \frac{\partial}{\partial t'} + \beta \frac{\partial}{\partial t'}\right) \frac{\partial u'}{\partial t'} = \left(1 + \theta \frac{\partial}{\partial t'}\right) \left[ \frac{\partial^2 u'}{\partial r'^2} + \frac{1}{r'} \frac{\partial u'}{\partial r'} \right] - \frac{\sigma B_0^2}{\rho} U'. \tag{9}$$

Equation (9) is valid when the magnetic field is fixed relative to the fluid. If the magnetic field is fixed relative to the moving cylinder, Eq. (9) can be replaced with the following equation;

$$\left(1 + \lambda \frac{\partial}{\partial t'} + \beta \frac{\partial}{\partial t'}\right) \frac{\partial u'}{\partial t'} = \left(1 + \theta \frac{\partial}{\partial t'}\right) \left[ \frac{\partial^2 u'}{\partial r'^2} + \frac{1}{r'} \frac{\partial u'}{\partial r'} \right] - \frac{\sigma B_0^2}{\rho} (U' - Ut'^n). \tag{10}$$

Equations (9) and (10) can be written together as follows;

$$\left(1 + \lambda \frac{\partial}{\partial t'} + \beta \frac{\partial}{\partial t'}\right) \frac{\partial u'}{\partial t'} = \left(1 + \theta \frac{\partial}{\partial t'}\right) \left[ \frac{\partial^2 u'}{\partial r'^2} + \frac{1}{r'} \frac{\partial u'}{\partial r'} \right] - M^2 \left(1 + \lambda \frac{\partial}{\partial t'} + \beta \frac{\partial}{\partial t'}\right) (U' - Kt'^n). \tag{11}$$

where,

$$K = \begin{cases} 0 & \text{When } B_0 \text{ is fixed relative to the fluid.} \\ 1 & \text{When } B_0 \text{ is fixed relative to the moving cylinder.} \end{cases}$$

The initial and boundary conditions for the problem are

$$u' = \begin{cases} 0 & a \leq r' \leq b & \text{and} & t' \leq 0 \\ 0 & \text{at} & r' = a & t' > 0. \\ ut'^n & \text{at} & r' = b & t' > 0 \end{cases} \tag{12}$$

The mathematical model described by Eq. (11) and (12) is the general representation of the unsteady flow formation for the MHD Couette flow of a Burger's fluid between two infinite concentric cylinders when the outer cylinder is moving with  $Ut'^n$ . We set out to find solution for the impulsive movement of the outer cylinder, which corresponds to  $n = 0$ ;

#### 3.1 Solution

Employing the non-dimensional quantities

$$t = \frac{t'\nu}{a^2}, \quad r = \frac{r'}{a}, \quad u = \frac{u'}{U}, \quad \lambda = \frac{\lambda'\nu}{a^2}, \quad \theta = \frac{\theta'\nu}{a^2}, \quad \beta = \frac{\beta'\nu}{a^2}, \quad M^2 = \frac{\sigma B_0^2 a^2}{\rho\nu}, \quad r^* = \frac{b}{a}. \tag{13}$$

we have the following dimensionless governing equation;

$$\left(1 + \lambda \frac{\partial}{\partial t} + \beta \frac{\partial}{\partial t}\right) \frac{\partial u}{\partial t} = \left(1 + \theta \frac{\partial}{\partial t}\right) \left[ \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} \right] - M^2 \left(1 + \lambda \frac{\partial}{\partial t} + \beta \frac{\partial}{\partial t}\right) (U - Kt^n) \tag{14}$$

With the dimensionless initial and boundary conditions

$$u = \begin{cases} 0, & a \leq r \leq b, & \text{and} & t \leq 0 \\ 0, & \text{at} & r = 1 & t > 0. \\ 1, & \text{at} & r = r^* & t > 0 \end{cases} \tag{15}$$

where  $M$  is the Hartmann number and  $r^*$  is the radius ratio. The solution of Eq. (14) subject to the dimensionless initial and boundary conditions (15) can then be obtained by the Laplace transform technique. Now, define the Laplace Transform  $\bar{u}(r, s) = \int_0^\infty u(r, t) e^{-st} dt$ , ( $s > 0$ ), where  $e^{-st}$  is kernel of the transform and  $S$  is the transformed variable. Taking Eq. (15) taking into account, Eq. (14) becomes;

$$(s + \lambda s^2 + \beta s^2) \bar{u} = (1 + \theta s) \left[ \frac{\partial^2 \bar{u}}{\partial r^2} + \frac{1}{r} \frac{\partial \bar{u}}{\partial r} \right] - M^2 (1 + \lambda s + \beta s) \left( \bar{u} - \frac{k}{s} \right). \tag{16}$$

Eq. (16) is reduced to a linear, non-homogeneous, ordinary differential equation of second order, having the form

$$\frac{d^2 \bar{u}}{dr^2} + \frac{1}{r} \frac{d \bar{u}}{dr} - \delta^2 \bar{u} = -\frac{M(1 + \lambda s + \beta s)}{s}. \tag{17}$$

Together with boundary conditions

$$\bar{u} = \begin{cases} 0 & \text{at} & r = 1 \\ 1/s & \text{at} & r = \lambda \end{cases} \tag{18}$$

Where  $\delta^2 = \frac{(s + \lambda s^2 + \beta s^2)}{(1 + \theta s)} + \frac{M^2(1 + \lambda s + \beta s)}{(1 + \theta s)}$ .

Eq. (17) represents the standard form of non-homogeneous Bessel equation and its solution is therefore given as;

$$\bar{u} = c_1 I_0(r\delta) + c_2 K_0(r\delta) + \frac{M^2 k(1 + \lambda s + \beta s)}{s \delta^2}. \tag{19}$$

where,  $I_0, K_0$  are the modified Bessel functions of first and second kind of order zero,

$$c_1 = \frac{1}{s [I_0(\delta r^*) K_0(\delta) - I_0(\delta) K_0(\delta r^*)]} \left[ K_0(\delta) + \frac{M^2 k(1 + \lambda s + \beta s)}{\delta^2} \{K_0(K_0 r^* - K_0(\delta))\} \right].$$

$$c_2 = \frac{1}{s [I_0(\delta r^*) K_0(\delta) - I_0(\delta) K_0(\delta r^*)]} \left[ \frac{M^2 k(1 + \lambda s + \beta s)}{\delta^2} \{I_0(\delta) - I_0(\delta r^*)\} - I_0(\delta) \right].$$

To obtain the inverse form of Equation (19) we use a numerical procedure used in an algorithm given by Tzou [18]. In this method, functions in the Laplace domain  $S$  can be inverted to the time domain  $t$  as follows ;

$$u(r, t) = \frac{e^{\epsilon t}}{t} \left[ \frac{1}{2} \bar{u}(r, \epsilon) + \text{Re} \sum_{\mathcal{N}} \bar{u}(r, \epsilon + \frac{ik\pi}{t}) (-1)^k \right]. \tag{20}$$

Where  $Re$  refers to the “real part of  $i = \sqrt{-1}$  is the imaginary number,  $\mathcal{N}$  is the number of terms used in the Riemann-sum approximation, and  $\epsilon$  is the real part of the Bromwich contour that is used in inverting Laplace

transforms. Its accuracy depends on the value of  $\epsilon$  and the truncation error dictated by  $\mathcal{N}$  and the value of  $\epsilon$  must be selected so that the Bromwich contour encloses all the branch points. For faster convergence, the quantity  $\epsilon t = 4.7$  gives the most satisfactory results.

### 3.2 Skin Friction

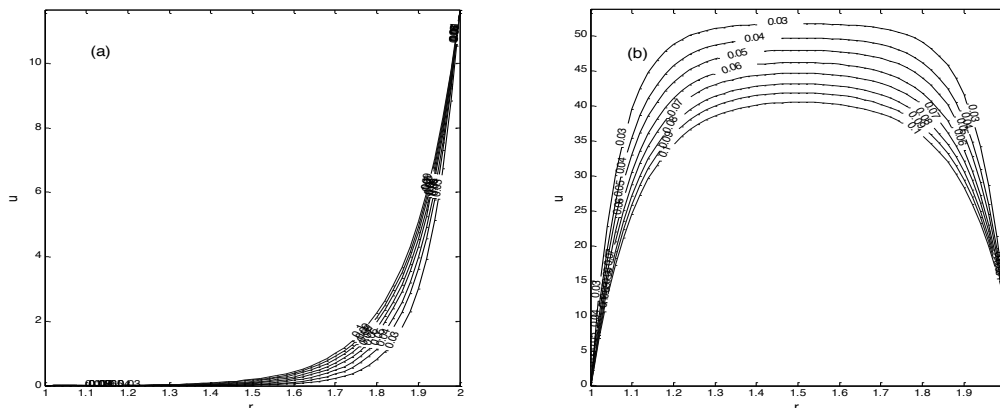
The skin friction (or shear stress)  $\tau_1(r, s)$  at the outer surface of the inner cylinder ( $r = 1$ ), and  $\tau_{r^*}(r, s)$  at the inner surface of the outer cylinder ( $r = r^*$ ), in terms of the Laplace parameter  $s$  is computed by differentiating Eq. (19).

$$\left. \frac{d\bar{u}}{dr} \right|_{r=1} = \delta [c_1 I_1(\delta) - c_2 K_1(\delta)] \tag{21}$$

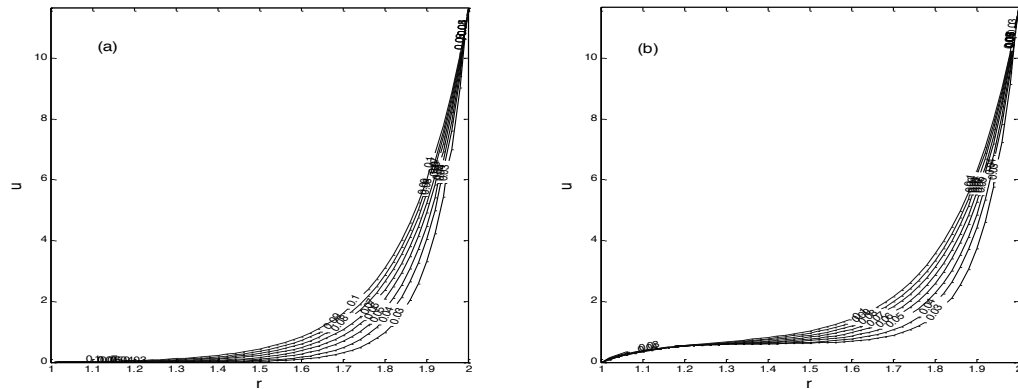
$$\left. \frac{d\bar{u}}{dr} \right|_{r=r^*} = \delta [c_1 I_1(\delta r^*) - c_2 K_1(\delta r^*)] \tag{22}$$

## 4. Results and Discussion

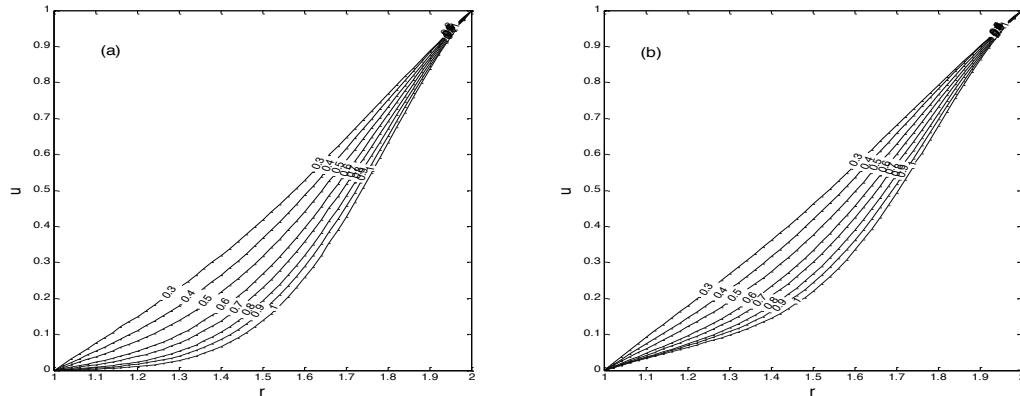
In order to get a physical insight into the problem, a MATLAB programme is written to compute and generate the graphs for the velocity, skin friction and mass flux in Figures 1 – 10 for various values of time ( $t$ ), Hartmann number ( $M$ ), relaxation time ( $\lambda$ ), retardation time ( $\theta$ ) and Burgers' parameter ( $\beta$ ) showing two cases when the magnetic field is fixed relative to the fluid ( $K = 0$ ) and when the magnetic field is fixed relative to the moving outer cylinder ( $K = 1$ ) represented by (a) and (b) respectively. The parametric values are arbitrarily chosen to study their effect on the flow domain. Figures 1 – 4 show the velocity profiles against  $r$ , depicting the effects of various non-dimensional parameters emerging in the flow model. Velocity is an increasing function of time and retardation time  $kr$ . It is decreasing function of relaxation times  $k$ ;  $k_1$  and kinematic viscosity. Shear stress is an increasing function of time and retardation time  $kr$ . It is decreasing function of relaxation times  $k$ ;  $k_1$  and dynamic viscosity.



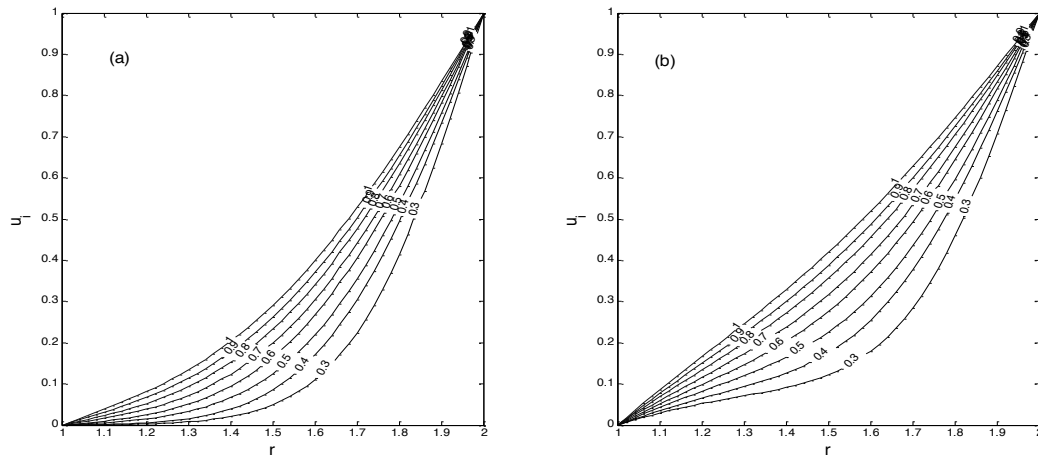
**Figure 1:** Velocity profiles against  $r$  for different values of  $t$  and  $M = 5$ ,  $\lambda = 0.02$ ,  $\theta = 0.02$ , with  $K = 0$  and  $K = 1$  represented by (a) and (b).



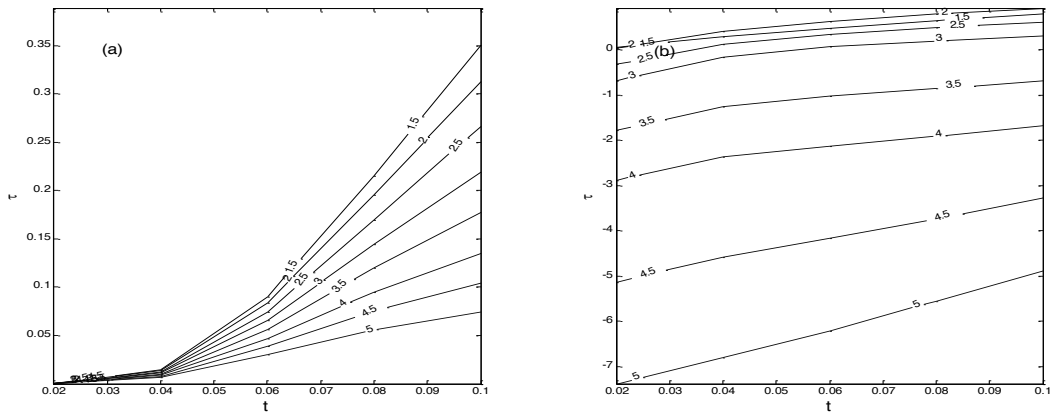
**Figure 2:** Velocity profiles against  $r$  for different values of  $t$  and  $M = 0.5$ ,  $\lambda = 0.2$ ,  $\theta = 0.2$  with  $K = 0$  and  $K = 1$  represented by (a) and (b).



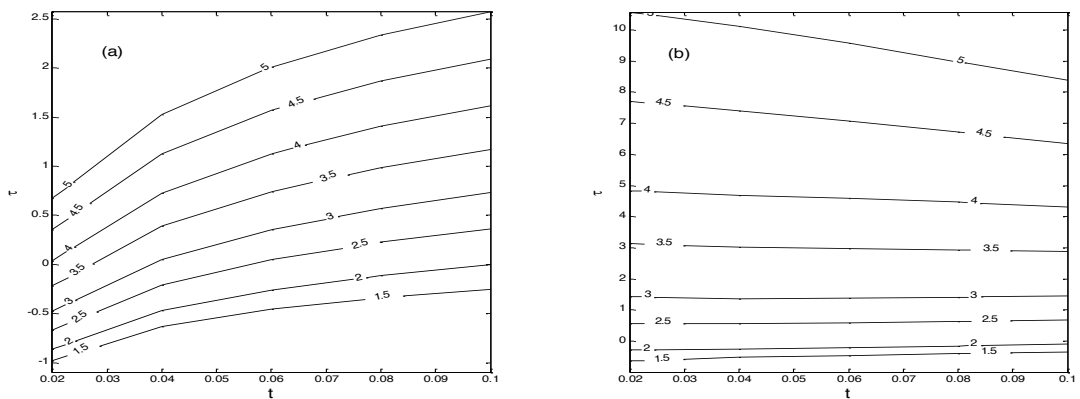
**Figure 3:** Velocity profiles against  $r$  for different values of  $\lambda$  and  $M = 0.5$ ,  $t = 0.2$ ,  $\theta = 0.2$  with  $K = 0$  and  $K = 1$ , represented by (a) and (b).



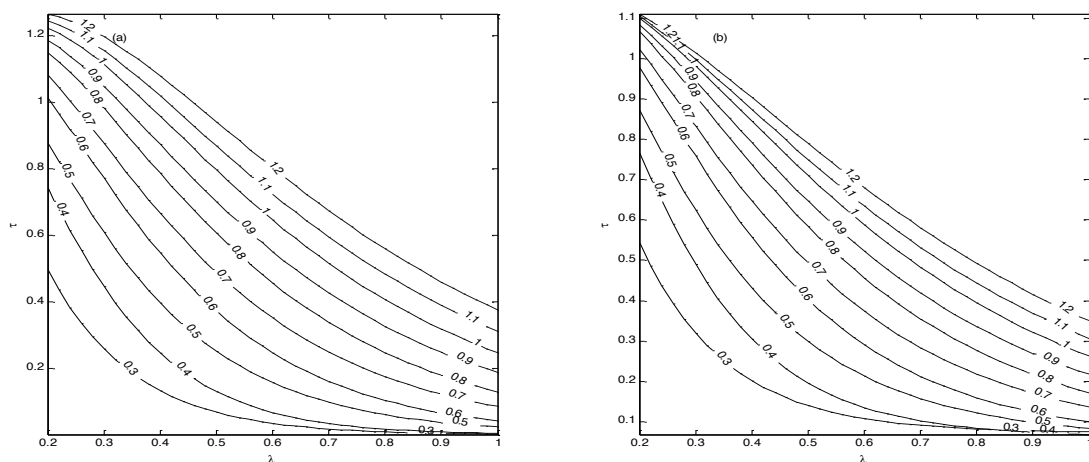
**Figure 4:** Velocity profile against  $r$  for different values of  $\theta$  and  $M = 0.5$ ,  $t = 0.02$ ,  $\lambda = 0.2$  with  $K = 0$  and  $K = 1$ , represented by (a) and (b).



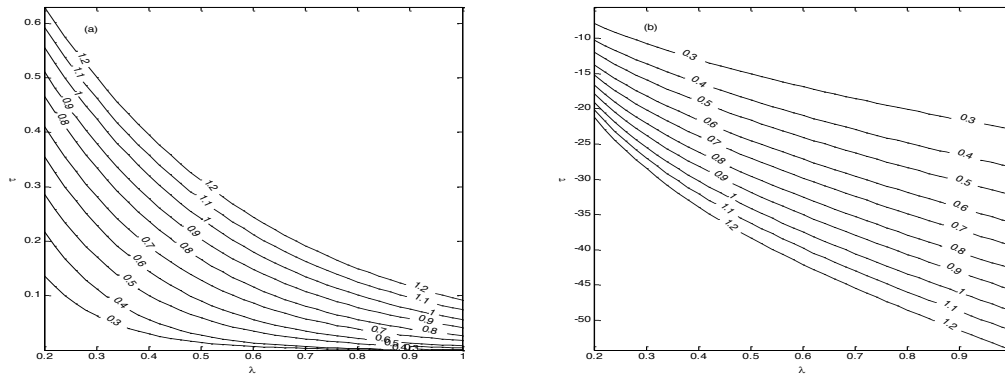
**Figure 5:** Variation of skin frictions against  $t$  on the outer surface of the inner cylinder ( $r = 1$ ) for different values of  $M$  and  $\lambda = 0.2, \theta = 0.2$  with  $K = 0$  and  $K = 1$ , represented by (a) and (b).



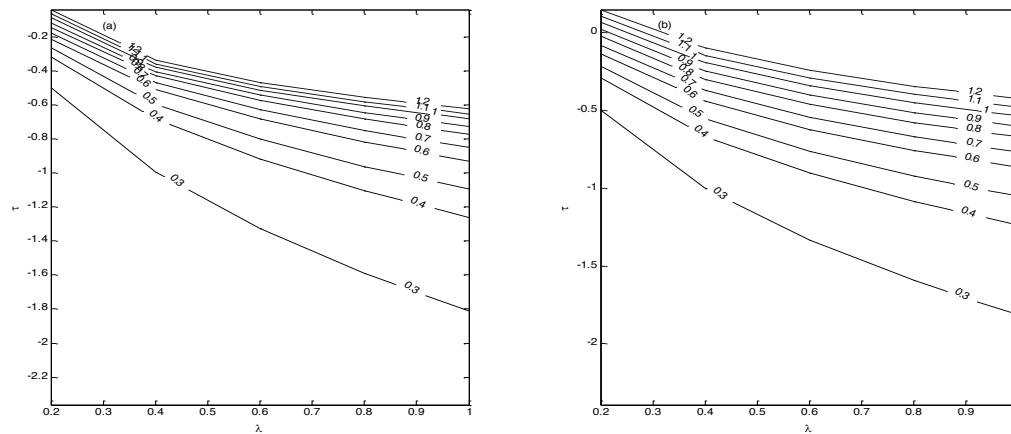
**Figure 6:** Variation of skin frictions against  $t$  on the inner surface of the outer cylinder ( $r = 2$ ) for different values of  $M$  and  $\lambda = 0.2, \theta = 0.2$  with  $K = 0$  and  $K = 1$ , represented by (a) and (b).



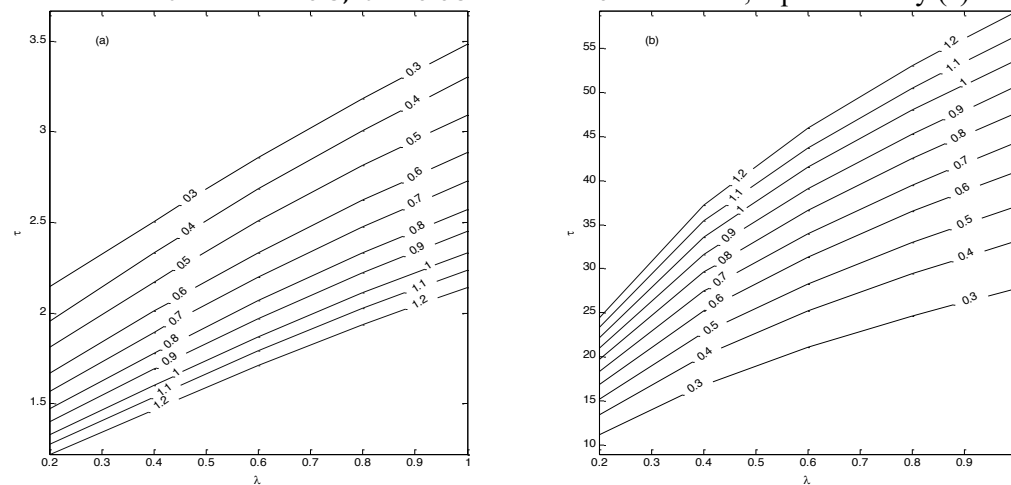
**Figure 7:** Variation of skin frictions against  $\lambda$  on the outer surface of the inner cylinder ( $r = 1$ ) for different values of  $\theta$  and  $M = 0.5, t = 0.08$  with  $K = 0$  and  $K = 1$ , represented by (a) and (b).



**Figure 8:** Variation of skin frictions against  $\lambda$  on the outer surface of the inner cylinder ( $r = 1$ ) for different values of  $\theta$  and  $M = 5.0$ ,  $t = 0.08$  with  $K = 0$  and  $K = 1$ , represented by (a) and (b).



**Figure 9:** Variation of skin frictions against  $\lambda$  on the inner surface of the outer cylinder ( $r = 2$ ) for different values of  $\theta$  and  $M = 0.5$ ,  $t = 0.08$  with  $K = 0$  and  $K = 1$ , represented by (a) and (b).



**Figure 10:** Variation of skin friction against  $\lambda$  on the inner surface of the outer cylinder ( $r = 2$ ) for different values of  $\theta$  and  $M = 5$ ,  $t = 0.08$  with  $K = 0$  and  $K = 1$ , represented by (a) and (b).



## 5. Conclusions

In conclusion, semi analytical expressions for velocity and the associated skin friction (shear stress) for the MHD Couette flow of Burger's fluid in annuli are obtained. The expressions are given in Laplace transform domain with the aid of modified Bessel equations while the inverse of Laplace transform by using Riemann sum approximation as mentioned above. Impact of various physical parameters on the fluid velocity has been graphically presented. Significant features of the current analysis are as follows; (i) Increase in relaxation time ( $\theta$ ) increases the velocity. While increase in retardation time ( $\lambda$ ) decreases the velocity. (ii) Increase in  $M$  decreases the velocity and the skin friction on the outer surface of the inner cylinder while the reverse is case on the inner surface of the outer cylinder.

## References

- Alsaedi, A., Alsaedi, F., E., Ali, S., & Hayat, T. (2013). Stagnation point flow of Burgers' fluid and Mass Transfer with chemical reaction and porosity. *Journal of Mechanics*, 29, (3), 453-460.
- Ahmad, I., Ali, N., Abbasi, A., Aziz, W., Hussain, M., Ahmad, M., Taj, M., & Zaman, Q., (2014). Flow of a Burger's Fluid in a Channel Induced by Peristaltic Compliant Walls. *Journal of Applied Mathematics*. <http://dx.doi.org/10.1155/2014/236483>.
- Burgers, J.M. (1935). Mechanical Considerations Model Systems Phenomenological Theories of relaxation and of viscosity, in: J.M. Burgers (Ed.), First Report on Viscosity and Plasticity, Nordemann Publishing Company, New York.
- Chen, C. I., Hayat, T., & Chen, J. (2006). Exact solutions for the unsteady flow of a Burger's fluid in a duct induced by time-dependent prescribed volume flow rate. *Heat Mass Transfer*, 43, 85–90.
- Chen, C., I., Chen, C.K., & Yang, Y.T., (2004). Unsteady unidirectional flow of an Oldroyd-B fluid in a circular duct with different given volume flow rate conditions. *Heat and Mass Transfer*, 40, 203–209. DOI 10.1007/s00231-002-0350-7
- Chun-I, C., Tasawar, H., & Jiann-Lin, C., (2006). Exact solutions for the unsteady flow of a Burger's fluid in a duct induced by time-dependent prescribed volume flow rate. *Heat Mass Transfer*, 43, 85–90. DOI 10.1007/s00231-006-0092-z.
- Constantin, F., & Dumitru Vieru, (2021). Symmetric and Non-Symmetric Flows of Burgers' Fluids through Porous Media between Parallel Plates Symmetry, 13, 1109. <https://doi.org/10.3390/sym13071109>
- Ersoy, H.V. (1999). MHD flow of an Oldroyd-B fluid due to non-coaxial rotations of a porous disk and the fluid at infinity. *International Journal of Engineering Science*, 37, 1973–1984.
- Fetecau, C., Hayat, T., Khan, M., & Fetecau, C. (2011). Erratum to: Unsteady flow of an Oldroyd-B fluid induced by the impulsive motion of a plate between two side walls perpendicular to the plate. *Acta Mechanica*, 216, 359–361.
- Hayat, T., Hutter, K., Asghar, S., & Siddiqui, A.M. (2002). MHD flows of an Oldroyd-B fluid. *Math Comp Model*. 36:987–995
- Hayat, T., Khan, M., & Ayub, M., (2004). Exact solutions of flow problems of an Oldroyd-B fluid. *Appl. Math Comput*, 151, 105–119.
- Hayat, T., Nadeem, S., & Asghar, S. (2004). Hydromagnetic Couette flow of an Oldroyd B fluid in a rotating system. *International Journal of Engineering Science*, 42, 65–78.
- Hyder, S., & Shah, A.M., (2010). Some Helical flows of a Burgers' fluid with fractional derivative. *Meccanica* 45, 143–151. DOI 10.1007/s11012-009-9233-z.
- Jamil, M., Fetecau, C., & Rana, M. (2012). *J. Theo. Applied Mech.* 50, 2, 562
- Jianhong, K., Yingke, L., & Tongqiang Xia., (2015). Unsteady Flows of a Generalized Fractional Burgers' Fluid between Two Side Walls Perpendicular to a Plate. *Advances in Mathematical Physics*, <http://dx.doi.org/10.1155/2015/521069>.
- Khan, M., Arshad, M., Anjum, A. (2012). On exact solutions of Stokes second problem for MHD Oldroyd-B fluid. *Nuclear Engineering Design*, 243(), 20–32.

- Kashif A. A., Mukarrum H., & Mirza M.B., (2018). A Mathematical Analysis of Magnetohydrodynamic Generalized Burger Fluid for Permeable Oscillating Plate. *Punjab University Journal of Mathematics* 50(2), 97-111.
- Khan, M., Malik, R., & Anjum, A. (2011). Analytical solutions for MHD flows of an Oldroyd-B fluid between two side walls perpendicular to the plate. *Chemical Engineering Communication*, 198(), 1415–1434.
- Khan M., Asia, A., & Fetecau, C. (2010). On exact solutions of Stokes second problem for a Burgers' fluid, I. The case  $\gamma < \lambda/4$ . *Z. Angew. Math. Phys.* 61, 697–720. DOI 10.1007/s00033-009-0025-z.
- Krishnan, J.M., & Rajagopal, K.R. (2004). A thermodynamic framework for the constitutive modeling of asphalt concrete: theory and application. *Journal of Material Civil Engineering*, 16(), 155–166.
- Krishnan, J.M., Rajagopal, K.R. (2003). Review of the uses and modeling of bitumen from ancient to modern times. *Appl. Mech. Rev.*, 56(), 149–214
- Liu, Y., Zheng, L., Zhang, X. (2011). Unsteady MHD Couette flow of a generalized Oldroyd-B fluid with fractional derivative. *Comput. Math. Appl.*, 61, 443–450.
- Madeeha, T., Muhammad, N.N., Maria, J., Muhammad, Y., Muhammad, I., Naeem, S., and Rabia, S., (2018). Unsteady flow of fractional Oldroyd-B fluids through rotating annulus. *Open Phys.*, 16, 193–200. <https://doi.org/10.1515/phys-2018-0028>
- Maria, J., Madeeha, T., Muhammad, I., Dumitru, B., Ali, A.I., Muhammad A.I., (2022). Unsteady flow of fractional Burgers' fluid in a rotating annulus region with power law kernel. *Alexandria Engineering Journal*, 61, 17–27.
- Masood K., (2013). Stokes' First Problem for an MHD Burgers Fluid. *Commun. Theor. Phys.* 59 (1), 99–104.
- Musa, A., Isah A., Gulibur D.Y., Garba, Adamu T., Mohammed, A., & Ali, M.B., (2023). Transient MHD flow of fractional Burger's fluid model through a parallel microchannel with heat transfer and slip boundary condition. *J. Process Mechanical Engineering*, 1–14. <https://DOI: 10.1177/09544089221146853>
- Nirmala, P., Saila, H., & Kumari, A., (2019). Unsteady and steady non-linear state flow of Burgers' fluid in the presence of magnetic field with heat generation/absorption. *Bull. Pure Appl. Sci. Sect. E Math. Stat.*, 38E (2), 570–585. DOI: 10.5958/2320-3226.2019.00058.4.
- Pai, S. I. (1962). *Magnetogasdynamics and Plasma Dynamics* (Springer, Berlin).
- Parag R., Murali, K., & J., Rajagopal, K., R., (2004). A note on the flow of a Burgers fluid in an orthogonal Rheometer. *International Journal of Engineering Science*, 42, 1973–1985.
- Quintanilla, R., & K., R., Rajagopal, (2012). Further mathematical results concerning Burgers fluids and their generalizations. *Z. Angew. Math. Phys.*, 63, 191–202. DOI 10.1007/s00033-011-0173-9
- Ravindran, P., Krishnan, J.M., & Rajagopal, K. R. (2004). A note on the flow of a Burgers' fluid in an orthogonal rheometer. *International Journal of Engineering Science*, 42, 1973–1985.
- Sani I., & Jha, B.K., (2019). Unsteady MHD Couette flow of Oldroyd-B-fluid in annuli: A semi-analytical approach. *BAOBAB: A Journal in Research and Science Technology*, 1(2), 116-128.
- Sarpkaya, T. (1961). Flow of non-Newtonian fluids in a magnetic field. *AICHEJ* 7, 324 – 328.
- Tzou, D., Y. Macro to Microscale Heat Transfer: The Lagging Behaviour (Taylor and Francis. London. 1997).
- Yan, Z., Hao, J., Z., & Yu, B., (2017). MHD Flow and Heat Transfer of a Generalized Burgers' Fluid Due to an Exponential Accelerating Plate with Effects of the Second Order Slip and Viscous Dissipation. *Commun. Theor. Phys.*, 67(6), 697–703.
- Yaqing, L., Liancun, Z., & Xinxin Z. (2011). MHD flow and heat transfer of a generalized Burgers' fluid due to an exponential accelerating plate with the effect of radiation. *Computers and Mathematics with Applications*, 62, 3123–3131.
- Zheng, L., Liu Y., & Zhang, X. (2012). Slip effects on MHD flow of a generalized Oldroyd-B fluid with fractional derivative. *Nonlinear Anal Real World Application*, 13, 513–523.
- Zheng, L., Liu Y., & Zhang, X. (2011). Exact solutions for MHD flow of generalized Oldroyd-B fluid due to an infinite accelerating plate. *Math Computational Modelling*, 54, 780–788.