

# Inventory Ordering Policies of Delayed Deteriorating Items with Ramp Type Demand and Price Discount

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## Abstract

*This study presents the inventory ordering policies of delayed/ non- instantaneous deteriorating items with Ramp Type demand and Price discount. The Ramp Type demand rate is very common when some brand of consumer goods is taken to the market and subsequently, their demand increases linearly at the beginning and then the market grows into a stable stage such that the demand becomes a constant up to the end of the inventory cycle. The assumption of price discount on selling price for the proposed model serves as a motivational factor to the wholesalers and retailers alike which in turn is instrumental to the significant increment in demand for the items put up for sales. In this situation a discount on selling price provides significant increment in demand and also maximize profit. The proposed model is developed incorporating both pre and post deteriorating discount on unit selling price and solutions of the model is provided analytically. A suitable iterative procedure is employed to solve the mathematical solutions in order to come up with numerical examples on the applications of the model. Sensitivity analysis would be carried out to bring to the fore, the effect of changes in the solutions to the model occasioned by slight changes in the system parameters.*

**Keywords:** Delayed deteriorating, Inventory, Ramp Type demand, Price discount.

## 1. Introduction

All organizations keep inventory; which includes a company's raw materials, work in progress, supplies used in operations, and finished goods. The term inventory refers to the goods or materials used by a company for the purpose of production and sale. In the industries, firms, factories, and markets, inventory models play a very important role. Perishability of products is an important aspect of inventory control. Deterioration, in general may be considered as a result of various effect on stock, some of which are damage, spoilage, decay, decreasing usefulness and many more. Goods stored undergo forms of gradual loss of potential ability with the passage of time, grain, electronics goods, radioactive substances also deteriorate.

The decaying inventory problem was first analyzed by Ghare and Shrader (1963) who developed EOQ model with constant rate of decay. Covert and Philip (1973) generalized Ghare and Shrader (1963) model by using two parameter weibull distribution. Later it was generalized by Tadikmalla (1978) using three-parameter gamma distribution taken to represent the time for deterioration. Hollier and Mark (1983) developed a model for inventory replenishment policies for deteriorating items in a declining market. Cheng and Chen (2004) considered deterioration items in periodic review environment with shortages.

In the classical EOQ model Harris (1915) assumed a constant demand rate, but Donaldson (1977) extended it for linear trend demand rate. Hill (1995) developed the concept of Ramp-type demand. In Ramp-type demand rate, when a new brand of consumer goods comes to the market, demand rate increases at the beginning of the season up to a certain time period say  $\mu$  and then remain to be constant for the rest of the time. The Ramp-type demand rate

has been used for those types of items, such as new brand of consumer goods and so on, whose demand is increased for a few times period and then decreases.

Every organization dealing with inventory faces a number of fundamental problems. Pricing decision is one of them. It has to decide how much to ask for each to unit and when to review the price as the season rolls on. It always wants the price to be marginal, not so high that it put off potential buyer and not so low that it losses out potential profits. Therefore, price can be considered as an important tool to influence demand. This led many researchers to investigate pricing strategy on inventory models in details Arcelus and Srinivasan (1998), Shah and Shah (1993), Wee and Law (2001). Khouja (2000) studied an inventory problem under the condition that multiple discounts can be used to sell excess inventory. The nature of independent effects of temporary price discounts on inventory policies is documented in literature (Weatherford and Bodily 1992; Petruzzi and Dada 1999) considering various assumptions. Neff (2000) mentioned that discount is considered as a way of boosting sales. The resulting increase in demand is expected to generate higher revenues and to accelerate inventory depletion rates with corresponding decrease in holding cost.

## 2. Assumptions

- (1) Replenishment is instantaneous.
- (2) Shortages are not allowed.
- (3) The demand  $d_2$  is assumed to be ramp type function of  $t$ .

$$\text{Where, } d_2 = \begin{cases} D_0 t, & t < \mu \\ D_0 \mu, & t \geq \mu \end{cases}$$

Where  $\mu$  is the changing point from linear demand to the constant demand.

- (4) During  $0 \leq t \leq T_1$  discount of  $r_1$  and effect on the discounted selling price on the demand before deterioration set in is  $\alpha_1 = (1 - r_1)^{-n_1}$ , where  $n_1 \in R$ .  
 During  $T_1 \leq t \leq T$  discount of  $r_2$  and effect on the discounted selling price on the demand when deterioration begins is  $\alpha_2 = (1 - r_2)^{-n_2}$ .

## 3. Notations

- $d_1$  Demand rate (unit per unit time) during the period before deterioration sets in.
- $d_2$  Demand rate (unit per unit time) after deterioration sets in.
- $\theta$  Rate of deterioration.
- $r_1$  Discount offer per unit before deterioration.
- $r_2$  Discount offer per unit when deterioration is set in.
- $T$  Inventory cycle length (per unit time).
- $I(t)$  Inventory level at time  $[0, T_1]$
- $I_d(t)$  Inventory level at time  $[T_1, T]$
- $T_1$  Time deterioration sets in.
- $h$  Holding cost per unit.
- $C$  unit cost of an item
- $C_0$  Cost of a unit deteriorating item.
- $S$  Unit selling price of an item.
- $A_0$  Ordering cost
- $I_0$  Initial inventory

4. Model formulation

Figure 1 illustrates the variation of Inventory level at time.

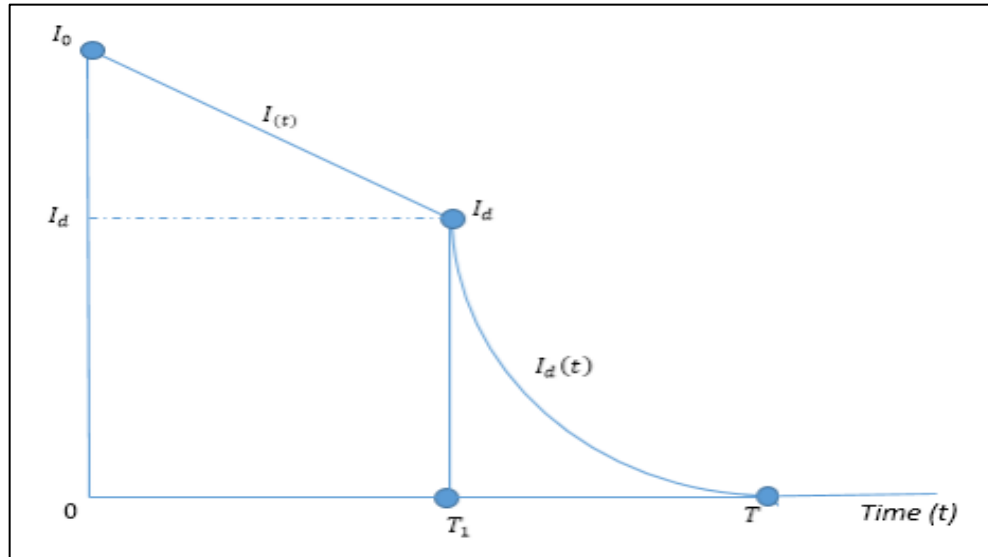


Figure 1: Variation of Inventory with time

$$\frac{dI(t)}{dt} = -d_1\alpha_1; \quad 0 \leq t \leq T_1. \tag{1}$$

$$\frac{dI_d(t)}{dt} + \theta I_d(t) = -d_2\alpha_2; \quad T_1 \leq t \leq T \tag{2}$$

where  $d_2 = \begin{cases} D_0t, & t < \mu \\ D_0\mu, & t \geq \mu \end{cases}$

Solution of Equation (1) is:  $I(t) = -d_1\alpha_1(t - T_1) + I_d$  (3)

Also, the solution of Equation (2) for  $t < \mu$ ,  $d_2 = D_0t$

$\frac{dI_d(t)}{dt} + \theta I_d(t) = -D_0t\alpha_2; \quad T_1 \leq t \leq \mu$  is as follows (4)

$$I_d(t) = -D_0\alpha_2 \left[ \frac{t}{\theta} - \frac{1}{\theta^2} + \frac{1}{\theta^2} e^{\theta(T_1-t)} - \frac{T_1 e^{\theta(T_1-t)}}{\theta} \right] + I_d e^{\theta(T_1-t)}$$

Solution of Equation (2) for  $t \geq \mu$ ,  $d_2 = D_0\mu$ ,  $\frac{dI_d^*(t)}{dt} + \theta I_d = -D_0\alpha_2\mu$ , and  $\mu \leq t \leq T$ .

$$I_d^*(t) = D_0\alpha_2 \frac{\mu}{\theta} [e^{\theta(T-t)} - 1] \tag{5}$$

4.1 Holding Cost

$$HC = h \left[ \int_0^{T_1} I(t) dt + \int_{T_1}^T I_d^*(t) dt \right] \tag{6}$$

where  $\int_{T_1}^T I_d^*(t) dt = \int_{T_1}^{\mu} I_d(t) dt + \int_{\mu}^T I_d^*(t) dt$

$$HC = h \left[ \int_0^{T_1} (-d_1 \alpha_1 t + I_0) dt + \int_{T_1}^{\mu} \left( -D_0 \alpha_2 \left[ \frac{t}{\theta} - \frac{1}{\theta^2} + \frac{1}{\theta^2} e^{\theta(T_1-t)} - \frac{T_1 e^{\theta(T_1-t)}}{\theta} \right] + I_d e^{\theta(T_1-t)} \right) dt + \int_{\mu}^T \left( D_0 \alpha_2 \frac{\mu}{\theta} [e^{\theta(T-t)} - 1] \right) dt \right] \tag{7}$$

$$HC = h(A + B + C) \tag{8}$$

where  $A = \int_0^{T_1} (-d_1 \alpha_1 t + I_0) dt$ . Substitute,  $I_0 = I_d + d_1 \alpha_1 T_1$  in  $A$

$$A = \int_0^{T_1} (-d_1 \alpha_1 t) dt + \int_0^{T_1} (I_d + d_1 \alpha_1 T_1) dt$$

$$A = -d_1 \alpha_1 \int_0^{T_1} (t) dt + (I_d + d_1 \alpha_1 T_1) \int_0^{T_1} dt$$

$$A = \frac{1}{2} (d_1 \alpha_1 T_1^2) + I_d T_1 \tag{9}$$

$$B = \int_{T_1}^{\mu} \left( -D_0 \alpha_2 \left[ \frac{t}{\theta} - \frac{1}{\theta^2} + \frac{1}{\theta^2} e^{\theta(T_1-t)} - \frac{T_1 e^{\theta(T_1-t)}}{\theta} \right] + I_d e^{\theta(T_1-t)} \right) dt$$

$$B = -D_0 \alpha_2 \left[ \left( \frac{\mu^2}{2\theta} - \frac{\mu}{\theta^2} - \frac{1}{\theta^3} e^{\theta(T_1-\mu)} + \frac{T_1 e^{\theta(T_1-\mu)}}{\theta^2} \right) - \left( \frac{T_1^2}{2\theta} - \frac{T_1}{\theta^2} - \frac{1}{\theta^3} + \frac{T_1}{\theta^2} \right) \right] - \left( \frac{I_d}{\theta} e^{\theta(T_1-\mu)} - \frac{I_d}{\theta} \right) \tag{10}$$

$$C = \int_{\mu}^T \left( D_0 \alpha_2 \frac{\mu}{\theta} [e^{\theta(T-t)} - 1] \right) dt$$

$$C = D_0 \alpha_2 \frac{\mu}{\theta} \left[ \frac{1}{\theta} (e^{\theta(T-\mu)} - 1) + \mu - T \right] \tag{11}$$

$$\therefore HC = h(A + B + C)$$

$$HC = h \left[ \frac{1}{2} (d_1 \alpha_1 T_1^2) + I_d T_1 - D_0 \alpha_2 \left[ \left( \frac{\mu^2}{2\theta} - \frac{\mu}{\theta^2} - \frac{1}{\theta^3} e^{\theta(T_1-\mu)} + \frac{T_1 e^{\theta(T_1-\mu)}}{\theta^2} \right) - \left( \frac{T_1^2}{2\theta} - \frac{T_1}{\theta^2} - \frac{1}{\theta^3} + \frac{T_1}{\theta^2} \right) \right] - \left( \frac{I_d}{\theta} e^{\theta(T_1-\mu)} - \frac{I_d}{\theta} \right) + \frac{D_0 \alpha_2 \mu}{\theta} \left[ \frac{1}{\theta} (e^{\theta(T-\mu)} - 1) + \mu - T \right] \right] \tag{12}$$

#### 4.2 Purchasing Cost

$$PC = I_0 \times C, \text{ then } PC = (I_d + d_1 \alpha_1 T_1) C \tag{13}$$

#### 4.3 Total Selling Revenue

$$SR = S \left[ \alpha_1 (1 - r_1) \int_0^{T_1} d_1 dt + \alpha_2 (1 - r_2) \left[ \int_{T_1}^{\mu} D_0 t dt + \int_{\mu}^T D_0 \mu dt \right] \right]$$

$$SR = S \left[ d_1 \alpha_1 (1 - r_1) T_1 + \alpha_2 (1 - r_2) \left[ -\frac{\mu^2 D_0}{2} - \frac{T_1^2 D_0}{2} + D_0 \mu T \right] \right] \tag{14}$$

#### 4.4 Total Cost of deteriorating items (TCD)

$I_d$  = Total items demanded + deteriorating items.

Deterioration =  $I_d$  - Total items demanded during deterioration period. Then

$$\text{Deterioration} = I_d - \int_{T_1}^T d_2 dt$$

$$\text{But } \int_{T_1}^T d_2 dt = \int_{T_1}^{\mu} D_0 t dt + \int_{\mu}^T D_0 \mu dt$$

$$\therefore \int_{T_1}^T d_2 dt = \frac{D_0}{2} (2T\mu - T_1^2 - \mu^2)$$

Total number of items that deteriorated is

$$TD = I_d - \int_{T_1}^T d_2 dt$$

$$TD = -d_1 \alpha_1 T_1 + I_0 - \frac{D_0}{2} (2T\mu - T_1^2 - \mu^2) = I_0 - d_1 \alpha_1 T_1 - \frac{D_0}{2} (2T\mu - T_1^2 - \mu^2)$$

$$TCD = C_0 \left[ I_0 - d_1 \alpha_1 T_1 - \frac{D_0}{2} (2T\mu - T_1^2 - \mu^2) \right] \tag{15}$$

$$PF = \frac{1}{T} (SR - PC - HC - TCD - A_0)$$

$$PF = \frac{1}{T} \left[ S \left\{ d_1 \alpha_1 (1 - r_1) T_1 + \alpha_2 (1 - r_2) \left[ -\frac{\mu^2 D_0}{2} - \frac{T_1^2 D_0}{2} + D_0 \mu T \right] \right\} - (I_d + d_1 \alpha_1 T_1) C \right. \\ \left. - h \left[ \frac{1}{2} d_1 \alpha_1 T_1^2 + I_d T_1 - D_0 \alpha_2 \left( \frac{\mu^2}{2\theta} - \frac{\mu}{\theta^2} - \frac{1}{\theta^3} e^{\theta(T_1 - \mu)} + \frac{T_1 e^{\theta(T_1 - \mu)}}{\theta^2} - \left( \frac{T_1^2}{2\theta} - \frac{1}{\theta^3} \right) \right) \right. \right. \\ \left. \left. - \left( \frac{I_d}{\theta} e^{\theta(T_1 - \mu)} - \frac{I_d}{\theta} \right) + \frac{D_0 \alpha_2 \mu}{\theta} \left[ \frac{1}{\theta} (e^{\theta(T - \mu)} - 1) + \mu - T \right] \right] \right. \\ \left. - C_0 \left[ I_0 - d_1 \alpha_1 T_1 - \frac{D_0}{2} (2T\mu - T_1^2 - \mu^2) \right] - A_0 \right] \tag{16}$$

$$\begin{aligned} \frac{dPF}{dT} = & -\frac{1}{T^2} \left[ S \left\{ d_1 \alpha_1 (1 - r_1) T_1 + \alpha_2 (1 - r_2) \left[ -\frac{\mu^2 D_0}{2} - \frac{T_1^2 D_0}{2} + D_0 \mu T - D_0 \mu T \right] \right\} - (I_d + d_1 \alpha_1 T_1) C \right. \\ & - h \left[ \frac{1}{2} d_1 \alpha_1 T_1^2 + I_d T_1 - D_0 \alpha_2 \left( \frac{\mu^2}{2\theta} - \frac{\mu}{\theta^2} - \frac{1}{\theta^3} e^{\theta(T_1 - \mu)} + \frac{T_1 e^{\theta(T_1 - \mu)}}{\theta^2} - \left( \frac{T_1^2}{2\theta} - \frac{1}{\theta^3} \right) \right) \right. \\ & \left. \left. - \left( \frac{I_d}{\theta} e^{\theta(T_1 - \mu)} - \frac{I_d}{\theta} \right) + \frac{D_0 \alpha_2 \mu}{\theta} \left[ \frac{1}{\theta} (e^{\theta(T - \mu)} - 1) + \mu - T - T e^{\theta(T - \mu)} + T \right] \right] \right. \\ & \left. - C_0 \left[ I_0 - d_1 \alpha_1 T_1 - \frac{D_0}{2} (2T\mu - T_1^2 - \mu^2 - 2\mu T) \right] - A_0 \right] \end{aligned} \tag{17}$$

$$\begin{aligned} e^{\theta(T_1 - \mu)} \approx & 1 + \theta(T_1 - \mu), \text{ and } e^{\theta(T - \mu)} \approx 1 + \theta(T - \mu) \\ e^{\theta(T_1 - \mu)} = & -\frac{1}{T^2} \left[ S \left\{ d_1 \alpha_1 (1 - r_1) T_1 + \alpha_2 (1 - r_2) \left[ -\frac{\mu^2 D_0}{2} - \frac{T_1^2 D_0}{2} \right] \right\} - (I_d + d_1 \alpha_1 T_1) C \right. \\ & - h \left[ \frac{1}{2} d_1 \alpha_1 T_1^2 + I_d T_1 \right. \\ & - D_0 \alpha_2 \left( \frac{\mu^2}{2\theta} - \frac{\mu}{\theta^2} - \frac{1}{\theta^3} [1 + \theta(T_1 - \mu)] + \frac{T_1 [1 + \theta(T_1 - \mu)]}{\theta^2} - \left( \frac{T_1^2}{2\theta} - \frac{1}{\theta^3} \right) \right) \\ & - \left( \frac{I_d}{\theta} [1 + \theta(T_1 - \mu)] - \frac{I_d}{\theta} \right) \\ & \left. + \frac{D_0 \alpha_2 \mu}{\theta} \left[ \frac{1}{\theta} ([1 + \theta(T - \mu)] - 1) + \mu - T [1 + \theta(T - \mu)] \right] \right] \\ & \left. - C_0 \left[ I_0 - d_1 \alpha_1 T_1 - \frac{D_0}{2} (-T_1^2 - \mu^2) \right] - A_0 \right] \end{aligned} \tag{18}$$

$$\begin{aligned} = & -\frac{1}{T^2} \left[ S \left\{ d_1 \alpha_1 (1 - r_1) T_1 + \alpha_2 (1 - r_2) \left[ -\frac{\mu^2 D_0}{2} - \frac{T_1^2 D_0}{2} \right] \right\} - (I_d + d_1 \alpha_1 T_1) C \right. \\ & - h \left[ \frac{1}{2} d_1 \alpha_1 T_1^2 + I_d T_1 - D_0 \alpha_2 \left( \frac{\mu^2}{2\theta} - \frac{T_1 \mu}{\theta} + \frac{T_1^2}{2\theta} \right) - T_1 I_d + \mu I_d + D_0 \alpha_2 \mu [-T^2 + \mu T] \right] \\ & \left. - C_0 \left[ I_0 - d_1 \alpha_1 T_1 - \frac{D_0}{2} (-T_1^2 - \mu^2) \right] - A_0 \right] \end{aligned} \tag{19}$$

From Equation

$$I_0 = I_d + d_1 \alpha_1 T_1$$

$$\begin{aligned} \frac{dPF}{dT} = & -\frac{1}{T^2} \left[ S \left\{ d_1\alpha_1(1-r_1)T_1 + \alpha_2(1-r_2) \left[ -\frac{\mu^2 D_0}{2} - \frac{T_1^2 D_0}{2} \right] \right\} - (I_d + d_1\alpha_1 T_1)C \right. \\ & \left. - h \left[ \frac{1}{2} d_1\alpha_1 T_1^2 - D_0\alpha_2 \left( \frac{\mu^2}{2\theta} - \frac{T_1\mu}{\theta} + \frac{T_1^2}{2\theta} \right) + \mu I_d + \frac{D_0\alpha_2\mu}{\theta} [-\theta T^2 + \theta\mu T] \right] \right. \\ & \left. - C_0 \left[ I_d + d_1\alpha_1 T_1 - d_1\alpha_1 T_1 - \frac{D_0}{2} (-T_1^2 - \mu^2) \right] - A_0 \right] = 0 \end{aligned} \tag{20}$$

$$\begin{aligned} & hD_0\alpha_2\mu T^2 - hD_0\alpha_2\mu^2 T \\ & + \left[ S \left\{ d_1\alpha_1(1-r_1)T_1 + \alpha_2(1-r_2) \left[ -\frac{\mu^2 D_0}{2} - \frac{T_1^2 D_0}{2} \right] \right\} - (I_d + d_1\alpha_1 T_1)C \right. \\ & \left. - h \left[ \frac{1}{2} d_1\alpha_1 T_1^2 - D_0\alpha_2 \left( \frac{\mu^2}{2\theta} - \frac{T_1\mu}{\theta} + \frac{T_1^2}{2\theta} \right) + \mu I_d \right] - C_0 \left[ I_d - \frac{D_0}{2} (-T_1^2 - \mu^2) \right] - A_0 \right] = 0. \end{aligned} \tag{21}$$

$$aT^2 + bT + c = 0,$$

$$\text{where } a = hD_0\alpha_2\mu, \quad b = -hD_0\alpha_2\mu^2$$

$$\begin{aligned} \text{and } c = & \left[ S \left\{ d_1\alpha_1(1-r_1)T_1 + \alpha_2(1-r_2) \left[ -\frac{\mu^2 D_0}{2} - \frac{T_1^2 D_0}{2} \right] \right\} - (I_d + d_1\alpha_1 T_1)C \right. \\ & \left. - h \left[ \frac{1}{2} d_1\alpha_1 T_1^2 - D_0\alpha_2 \left( \frac{\mu^2}{2\theta} - \frac{T_1\mu}{\theta} + \frac{T_1^2}{2\theta} \right) + \mu I_d \right] - C_0 \left[ I_d + \frac{D_0 T_1^2}{2} + \frac{D_0 \mu^2}{2} \right] - A_0 \right] \end{aligned}$$

**Lemma 1:**

If  $h > 0$ ,  $D_0 > 0$ ,  $\alpha_2 > 0$ , and  $\mu > 0$ , then (i)  $a > 0$ , (ii)  $b < 0$ .

**Proof:**

(i) Since  $h > 0$ ,  $D_0 > 0$ ,  $\alpha_2 > 0$ , and  $\mu > 0$ , Then,  $a = hD_0\alpha_2\mu > 0$ .

(ii) From (i) above, also  $\mu^2 > 0$ ;  $hD_0\alpha_2\mu^2 > 0$ . Then,  $b = -hD_0\alpha_2\mu^2 = -a\mu < 0$ .

**Lemma 2:** If  $S < \frac{h \left[ \frac{1}{2} d_1\alpha_1 T_1^2 - D_0\alpha_2 \left( \frac{\mu^2}{2\theta} - \frac{T_1\mu}{\theta} + \frac{T_1^2}{2\theta} \right) + \mu I_d \right]}{\left\{ d_1\alpha_1(1-r_1)T_1 + \alpha_2(1-r_2) \left[ -\frac{\mu^2 D_0}{2} - \frac{T_1^2 D_0}{2} \right] \right\}}$ , then  $c < 0$ .

**Proof:**

$$\begin{aligned} c = & S \left\{ d_1\alpha_1(1-r_1)T_1 + \alpha_2(1-r_2) \left[ -\frac{\mu^2 D_0}{2} - \frac{T_1^2 D_0}{2} \right] \right\} - (I_d + d_1\alpha_1 T_1)C \\ & - h \left[ \frac{1}{2} d_1\alpha_1 T_1^2 - D_0\alpha_2 \left( \frac{\mu^2}{2\theta} - \frac{T_1\mu}{\theta} + \frac{T_1^2}{2\theta} \right) + \mu I_d \right] - C_0 \left[ I_d + \frac{D_0 T_1^2}{2} + \frac{D_0 \mu^2}{2} \right] - A_0 \end{aligned}$$

$$\begin{aligned}
 &< \frac{h \left[ \frac{1}{2} d_1 \alpha_1 T_1^2 - D_0 \alpha_2 \left( \frac{\mu^2}{2\theta} - \frac{T_1 \mu}{\theta} + \frac{T_1^2}{2\theta} \right) + \mu I_d \right]}{\left\{ d_1 \alpha_1 (1 - r_1) T_1 + \alpha_2 (1 - r_2) \left[ -\frac{\mu^2 D_0}{2} - \frac{T_1^2 D_0}{2} \right] \right\}} \left\{ d_1 \alpha_1 (1 - r_1) T_1 + \alpha_2 (1 - r_2) \left[ -\frac{\mu^2 D_0}{2} - \frac{T_1^2 D_0}{2} \right] \right\} \\
 &\quad - (I_d + d_1 \alpha_1 T_1) C - h \left[ \frac{1}{2} d_1 \alpha_1 T_1^2 - D_0 \alpha_2 \left( \frac{\mu^2}{2\theta} - \frac{T_1 \mu}{\theta} + \frac{T_1^2}{2\theta} \right) + \mu I_d \right] \\
 &\quad - C_0 \left[ I_d + \frac{D_0 T_1^2}{2} + \frac{D_0 \mu^2}{2} \right] - A_0 \\
 &= h \left[ \frac{1}{2} d_1 \alpha_1 T_1^2 - D_0 \alpha_2 \left( \frac{\mu^2}{2\theta} - \frac{T_1 \mu}{\theta} + \frac{T_1^2}{2\theta} \right) + \mu I_d \right] - (I_d + d_1 \alpha_1 T_1) C \\
 &\quad - h \left[ \frac{1}{2} d_1 \alpha_1 T_1^2 - D_0 \alpha_2 \left( \frac{\mu^2}{2\theta} - \frac{T_1 \mu}{\theta} + \frac{T_1^2}{2\theta} \right) + \mu I_d \right] - C_0 \left[ I_d + \frac{D_0 T_1^2}{2} + \frac{D_0 \mu^2}{2} \right] - A_0 \\
 &= -(I_d + d_1 \alpha_1 T_1) C - C_0 \left[ I_d + \frac{D_0 T_1^2}{2} + \frac{D_0 \mu^2}{2} \right] - A_0 < 0. \tag{22}
 \end{aligned}$$

**Theorem:** With conditions of Lemma (i) and (ii), the best value of  $T$  is  $T^* = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$

**Proof:** The quadratic equation,  $aT^2 + bT + c = 0$ .

Then if the 2 roots,  $T^* = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$  and  $T^{**} = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$

$b^2 - 4ac \geq 0$ , where  $b^2 > 0$  and  $b = b^2$ .

$b^2 < b^2 - 4ac$ ;  $b < \sqrt{b^2 - 4ac}$ ;  $b - \sqrt{b^2 - 4ac} < 0$ ;  $b < 0$ ,  $-b > 0$  and  $a > 0$ ,  $c < 0$ .

Then  $T^* = \frac{-b + \sqrt{b^2 - 4ac}}{2a} > 0$ , and  $T^{**} = \frac{-b - \sqrt{b^2 - 4ac}}{2a} < 0$ . Therefore,  $T^*$  is the best value of  $T$

$$\begin{aligned}
 \frac{d^2 PF}{(dT)^2} &= \frac{2}{T^3} \left[ S \left\{ d_1 \alpha_1 (1 - r_1) T_1 + \alpha_2 (1 - r_2) \left[ -\frac{\mu^2 D_0}{2} - \frac{T_1^2 D_0}{2} \right] \right\} - (I_d + d_1 \alpha_1 T_1) C - h \left[ \frac{1}{2} d_1 \alpha_1 T_1^2 - \right. \right. \\
 &\quad \left. \left. D_0 \alpha_2 \left( \frac{\mu^2}{2\theta} - \frac{T_1 \mu}{\theta} + \frac{T_1^2}{2\theta} \right) + \mu I_d + D_0 \alpha_2 \mu [-T^2 + \mu T] - C_0 \left[ I_d + \frac{D_0 T_1^2}{2} + \frac{D_0 \mu^2}{2} \right] - A_0 \right] + \right. \\
 &\quad \left. \frac{1}{T^2} [h D_0 \alpha_2 \mu^2 - 2h D_0 \alpha_2 \mu T] \right. \\
 &= \frac{2}{T^3} \left[ S \left\{ d_1 \alpha_1 (1 - r_1) T_1 + \alpha_2 (1 - r_2) \left[ -\frac{\mu^2 D_0}{2} - \frac{T_1^2 D_0}{2} \right] \right\} - (I_d + d_1 \alpha_1 T_1) C \right. \\
 &\quad \left. - h \left[ \frac{1}{2} d_1 \alpha_1 T_1^2 - D_0 \alpha_2 \left( \frac{\mu^2}{2\theta} - \frac{T_1 \mu}{\theta} + \frac{T_1^2}{2\theta} \right) + \mu I_d \right] - C_0 \left[ I_d + \frac{D_0 T_1^2}{2} + \frac{D_0 \mu^2}{2} \right] - A_0 \right] \\
 &\quad - \frac{h D_0 \alpha_2 \mu^2}{T^2} \\
 &= \frac{2c}{T^3} + \frac{b}{T^2} < 0 \tag{23}
 \end{aligned}$$



## 5. Results and Discussion

### 5.1 Numerical Example

The values of the following parameters are taken in appropriate units will be the same as that of Vandana (2018) and Shibaji et al. (2009). Let  $h = 30$ ,  $D_0 = 20000$ ,  $\mu = 0.25$ ,  $S = 45$ ,  $d_1 = 20$ ,  $C = 35$ ,  $I_d = 10$ ,  $C_0 = 40$ ,  $A_0 = 500$ ,  $\theta = 0.005$ ,  $r_1 = 0.01$ ,  $r_2 = 0.02$ ,  $T_1 = 0.27$ ,  $a = 154045.4$ ,  $b = -37983.8$ ,  $c = -71129$ , and substitute the values in  $T^*$ , Equations (6) and (23), we found the values of  $T^* = 0.81$  (year),  $I_0 = 15.59$  (units),  $PF = 248329.5$  (\$).

Table 1. Sensitivity analysis

Parameters	% Change in parameter	Change in $I_0$	Change in $T^*$	change in PF
h	0	15.5907	0.8139	248329.51
	-5	15.5907	0.8424	252859.92
	-20	15.5907	0.9408	266758.67
	20	15.5907	0.8129	298188.20
	5	15.5907	0.8136	260786.68
$D_0$	0	15.5907	0.8139	248329.51
	-5	15.5907	0.8142	235872.33
	-20	15.5907	0.8154	198500.79
	20	15.5907	0.9369	317467
	5	15.5907	0.9612	275089.74
$\mu$	0	15.5907	0.8139	248328.46
	-5	15.5907	0.4512	1448328.46
	-20	15.5907	0.4627	145271.31
	20	15.5907	0.8700	284723.36
	5	15.5907	0.8283	257414.49
S	0	15.5907	0.88138	248329.51
	-5	15.5907	0.7991	239225.10
	-20	15.5907	0.7527	212032.86
	20	16.7088	0.8139	248339.54
	5	15.5907	0.8139	248332.01
$d_1$	0	15.5907	0.8139	248329.51
	-5	15.5907	0.8140	248327.00
	-20	15.5907	0.8139	248319.48
	20	15.5907	0.8139	248329.35
	5	15.5907	0.8139	248329.47
$r_1$	0	15.5907	0.8139	248329.51
	-5	15.5907	0.8139	248329.55
	-20	15.5907	0.8139	248329.67
	20	15.5907	0.8106	248331.09
	5	15.5907	0.8131	248330.15
$r_2$	0	15.5907	0.8139	248329.51
	-5	15.5907	0.8147	248328.69
	-20	15.5907	0.8172	248325.28
	20	15.5907	0.4532	248131.31
	5	15.5907	0.4953	162667.73
$T_1$	0	15.5907	0.8135	248272.55
	-5	15.5907	0.9352	265389.53
	-20	15.5907	0.6949	265098.22

	20	15.5907	0.8147	248228.80
	5	15.5907	0.8141	248304.33
	0	15.5907	0.8139	248329.51
	-5	15.5907	0.8137	248354.68
$I_d$	-20	15.5907	0.8131	248430.20
	20	15.5907	0.8144	248250.79
	5	15.5907	0.8140	248309.83
	0	15.5907	0.8139	248329.51
	-5	15.5907	0.8138	248349.19
$C$	-20	15.5907	0.8133	248408.23
	20	15.5907	0.8497	251908.54
	5	15.5907	0.8243	249428.08
	0	15.5907	0.8139	248329.51
	-5	15.5907	0.8022	247036.40
$\theta$	-20	15.5907	0.7564	241318.55
	20	15.5907	0.8636	279935.95
	5	15.5907	0.8266	256220.92
	0	15.5907	0.8139	248329.51
	-5	15.5907	0.8009	240443.20
$C_0$	-20	15.5907	0.7604	216800.04
	20	15.5907	0.8144	248257.38
	5	15.5907	0.8140	248311.48
	0	15.5907	0.8139	248329.51
	-5	15.5907	0.8138	248349.54
$A_0$	-20	15.5907	0.8134	248401.64
	20	15.5907	0.7143	230292.17
	5	15.5907	0.7870	243822.94

From the Table 1, it is clear that the % values of  $h, D_0, \mu, S, T_1, C, C_0,$  and  $A_0$  of  $I_0$  values remain on change. The higher % values of  $d_1, r_1, r_2, I_d,$  of  $I_0$  values increase. The lower % values of  $d_1, r_1, r_2, I_d,$  of  $I_0$  values decrease. The higher % values of  $h, D_0, \mu, d_1, r_1,$  of  $T^*$  values decrease. Also, the higher % values of  $S, I_d, r_2, C_0, C, A_0, \theta$  of  $T^*$  values increase while the lower % values of  $h, D_0, \mu, d_1, r_1, T_1$  of  $T^*$  values increase. Furthermore, the lower % values of  $S, I_d, r_2, C_0, C, A_0, \theta$  of  $T^*$  values decrease. The higher % values of  $h, r_2, I_d, C, \theta, A_0$  of  $PF$  values decrease. The higher % values of  $D_0, \mu, S, r_1, d_1, C_0, T_1$  of  $PF$  values increase. The lower % values of  $h, r_2, I_d, C, \theta, A_0$  of  $PF$  values increase. The lower % values of  $D_0, \mu, S, r_1, d_1, C_0, T_1$  of  $PF$  values decrease.

## 6. Conclusion

In this paper, we present a mathematical model on the inventory of delayed deteriorating items (non- instantaneous), with ramp type demand. The Ramp Type demand rate is very common when some brand of consumer goods is taken to the market and subsequently, their demand increases linearly at the beginning and then the market grows into a stable stage such that the demand becomes a constant up to the end of the inventory cycle. The product which starts to deteriorate after a certain period of time, with components stock dependent demand pre- and post-deterioration discount are provided on the unit selling price of the product. When discounts are provided the demand is partially stock dependent and partially stock and selling price dependent. The mathematical model is developed allowing pre and post deterioration discount on the unit selling price. Then the amount of allowable both form of discounts improves the demand significantly and also maximize profit. Also, the proposed model can assist the manager to determine accurately the optimal order quantity and average cost per unit. Moreover, the proposed model can be used in inventory control of certain deteriorating items such as food items. In future work, it is possible to incorporate realistic assumptions such as probabilistic demand as a finite rate of replenishment.

## References

- Abubakar, M. and Babangida, S. (2012). Inventory ordering policies of delayed deteriorating items under permissible delay in payment. *Int. J. Production Economics*, Vol.136, Pp. 75-83.
- Arcelus, F.J. and Srinivasan, G. (1998). Ordering policies under one time discount and price sensitive demand. *IEEE Trans*, Vol.30, Pp. 1057-1064.
- Covert, R.P. and Phillip, G.C. (1973). An EOQ model for items with Weibull distribution. *American Institute of Industrial and Engineering Transaction*, Vol. 5, pp. 323-326.
- Donaldson, W.A. (1977). Inventory replenishment policy for linear trend in demand: an analytical solution. *Opl. Res. Q.* Vol. 28, pp. 663-670.
- Ghare, P.M and Shrader, G.F. (1963). A model for exponentially decaying inventories, *Journal of Industrial Engineering*, Vol. 14, pp. 238-243.
- Harris, F. (1915). How many parts to make at once, factory, *The Magazine of Management*, Vol. 10, pp. 135-136. 152 Reprinted in *Operational Research*, Vol. 38, No.6 (1990), pp.947-950.
- Khouja, M. (2000). Optimal ordering, discounting and pricing in a single-period problem. *International Journal of Production Economics*. Vol.65, pp.201-216.
- Levin, R.I., McLaughlin, C.P., Lamone, R.P., Kottas, J.F. (1972). *Productions/operations management contemporary policy for managing operating systems*. McGraw-Hill New York, p373.
- Neff, J. (2000). Trade promotion rises. *Advert Age* 71, pp.24-28.
- Peter, S.D., Robert, H.J., Peter, C. (2007). A note on the inventory models for deteriorating items with Ramp type demand rate. *European Journal of Operational Research*, Vol. 178, pp. 112-120.
- Petruzzi, N.C., Dada, M. (1999). Pricing and the news Vendor problem: a review with extensions. *Oper Res*, Vol. 47, pp. 183-194.
- Shah, N.H. and Shah, Y.K. (1993). An EOQ Model for Exponentially Decaying Inventory under Temporary Price Discounts. *Cahiers du CERO*. 35:227-232.
- Shibaji, P., Subrata, S., Manjusri B. (2009). An EOQ model for perishable products with discounted selling price and stock dependent demand. *Central European Journal of Operational Research*, Vol. 17, Pp. 31-53.
- Silver, E.A., and Peterson, R. (1985). *Decision system for inventory management and production planning*, 2nd edition. Wiley, New York.
- Skouri, K., Konstantaras, I., Papachristos, S. and Ganas, I. (2009). Inventory Model with Ramp type demand rate, partial backlogging and Weibull deterioration rate. *European Journal of Operational Research*, Vol. 12, No. 1, pp.79-92.
- Tadikamalla, P.R. (1978). An EOQ model for items with Gamma distribution. *AIIE Transaction*, Vol. 5, pp. 100-103.
- Vandana. (2018). Analysis of an Inventory Model with Time-dependent Deterioration and Ramp-type Demand Rate: Complete and Partial Backlogging. *An International Journal of Application and Applied Mathematics*, Vol. 13, pp.1076-1092.
- Weatherford, L.R. and Bodily, S.E.(1992). A taxonomy and research overview of perishable asset revenue management: yield management, overbooking, and pricing. *Operational Research*, Vol. 40, pp. 830-844
- Wee, H.M. and Law, S.T. (2001). Replenishment and pricing policy for deteriorating items taking into account the time-value of money. *International Journal of Production Economics*, Vol. 71, pp. 213-220.
- Wuu Wu J., Lin C., Tan B., Lee W.C. (1999). An EOQ inventory model with ramp type demand rate for items with Weibull deterioration, *Information and Management Science*, Vol. 10, pp.41-51